MATHEMATICS
PERFORMANCE DESCRIPTORS

GRADES 1-5

RESPONDING TO THIS DOCUMENT

We welcome your response to this document.

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INTRODUCTION

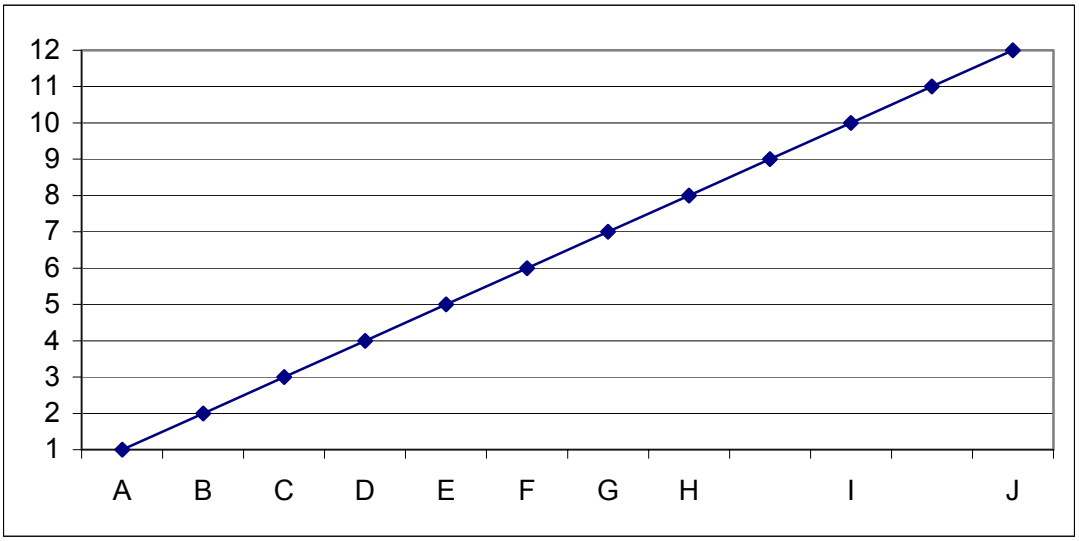
Design for Performance Standards

The Illinois Learning Standards are content standards that describe *"what"* students should know and be able to do in grades K – 12. Each content standard includes five benchmarks that describe what students should know and be able to do at early elementary, late elementary, middle/junior high, early high school, and late high school.

The challenge for the 2000-2001 school year was to produce performance standards that would indicate *"how well"* students should perform to meet the standards. To address this challenge a number of perspectives needed to be considered. For example, the National Governors Association[1] raised two pertinent questions policymakers should consider for the design of performance standards:

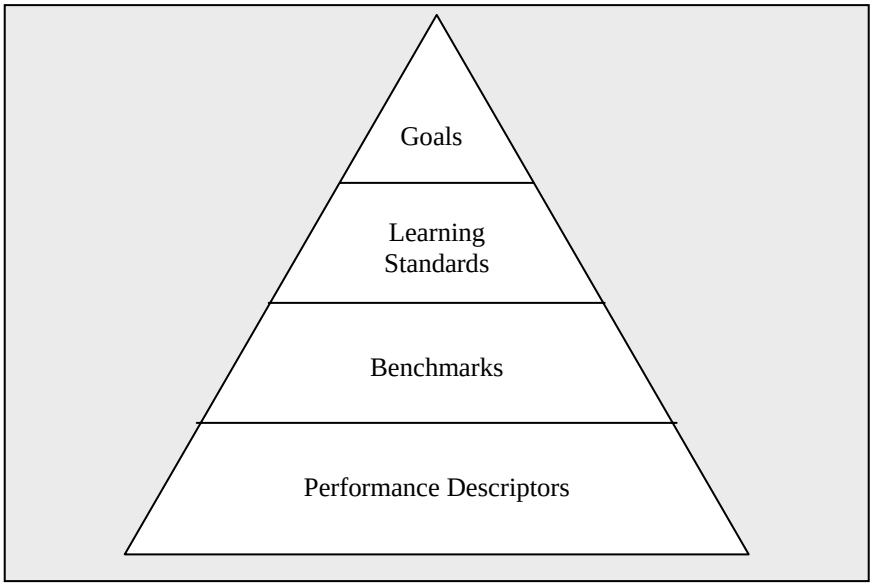
- Do the performance standards indicate the levels of performance students should attain, descriptions of performance at each level, and rules that enable educators to determine whether students have reached a given level?
- Do the performance standards include a range of work . . . to show that students can meet the standards in a variety of ways?

The performance standards describe how well students perform at various points on an educational development continuum. This continuum shows how students can demonstrate mastery of progressively more difficult content and cognitive skills over ten incremental stages of development. Performance within each stage can be assessed by the extent to which students are meeting the standards (i.e., starting, approaching, meeting, exceeding). Performance standards include four essential elements: performance descriptors, performance levels, assessment tasks, and performance examples.



[1] Ouellette, M. (2000). "Maintaining progress through systemic education reform: Performance standards," Washington, DC: National Governors Association.

The performance standards are classroom resources for voluntary use at the local level. They are not intended to replace the Illinois Learning Standards. Instead, they supplement them by providing sufficient detail and examples to enable teachers to establish appropriate grade-level performance expectations for students. The performance descriptors are a direct outgrowth of the state goals for learning. Whereas the benchmarks filled in detail on each of the standards at five grade-level clusters, the performance descriptors provide additional detail at each grade level.



Definitions

performance standards*:* the knowledge and skills that students are to perform at various stages of educational development (*performance descriptors*) and the performance expectations (*performance levels and assessment tasks)* for student work *(performance exemplars)* at each of the stages.

performance descriptors*:* statements of how students can demonstrate the knowledge and skills they acquired.[2]

performance levels*:* descriptions of how well students have achieved the standards; that is, the range, frequency, facility, depth, creativity, and/or quality of the knowledge and skills they acquired. Students can demonstrate levels of achieving performance standards along six dimensions:

PERFORMANCE LEVEL =	RANGE +	FREQUENCY +	FACILITY +	DEPTH +	CREATIVITY +	QUALITY
Exceeding	extensively	consistently	automatically	profoundly	inventively	excellently
Meeting	fully	usually	quickly	deeply	imaginatively	well
Approaching	partially	occasionally	haltingly	cursorily	commonly	marginally
Starting	narrowly	rarely	slowly	superficially	imitatively	poorly

[2] New Standards. Performance Standards. (1997) Washington, DC: The National Center on Education and the Economy.

assessment tasks: descriptions of what students can do to demonstrate they have met the standards and a means for evaluating the levels of their performance.

performance examples: student work samples resulting from the classroom-based assessment tasks that illustrate performance levels.

Template For Expanded Performance Descriptors

BENCHMARKS ⇒	early elementary			late elementary		middle/ junior high			early high	late high
STAGES ⇒ PERFORMANCE LEVELS ⇓	A	B	C	D	E	F	G	H	I	J
Exceeding										
Meeting										
Approaching										
Starting										

Vision for Mathematics Performance

Students need to see mathematics as a language, a tool, and an art form with which they can communicate ideas, solve problems, and explore the world around them. By the end of twelfth grade, they will have been encouraged to see multiple ways of expressing mathematical ideas, guided to make multiple connections to real life situations, and prompted to work with others as they explore possibilities. Their thinking will have become flexible, transferring mathematical knowledge from one situation to another, and they will be able to communicate their thinking in a variety of ways using any technology available to them.

Students will make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures. Through this process, they will develop mathematical reasoning skills and become adept at evaluating their own thinking. Students will have an understanding of how numbers are used and represented. They will be able to estimate and use basic operations to both solve everyday problems and confront more involved calculations in algebraic and statistical settings. They will be able to read, write, visualize, and talk about ways in which mathematical problems can be solved in both theoretical and practical situations. They will be able to communicate relationships in geometric and statistical settings through drawings and graphs.

Goal 6 – Demonstrate and apply a knowledge and sense of numbers, including numeration and operations (addition, subtraction, multiplication, division), patterns, ratios, and proportions. Students must have an understanding of and proficiency with counting, numbers and arithmetic, number systems and their structures. Development of number sense is central to the students' mathematical understanding. They must have the ability to compose, decompose, and recompose numbers, use the relationships among arithmetic operations to solve problems, and understand the base-ten number system. By the end of twelfth grade, students should be able to estimate, make sense of numbers, and recognize that the ability to determine relative and absolute magnitudes of numbers are important mathematical skills. The students must attain a rich understanding of the role of numbers: what they are; how they are

represented with objects, numerals, or on number lines; how they are related to one another; how numbers are embedded in systems that have structures and properties; and how to use numbers and operations to solve problems. Computational fluency of basic number combinations is essential, especially knowing the best tools to use for estimating and computing in a variety of situations: mental math, paper and pencil, and technology. The students must be able to explain their methods, understand that many methods exist, and see the usefulness of methods that are efficient, accurate, and applicable.

Goal 7 – Estimate, make, and use measurements of objects, quantities, and relationships and determine acceptable levels of accuracy. Students must develop an understanding of what is a measurable attribute and become familiar with the units and processes that are used in measuring. The tools, techniques, and formulas used in measuring must be developed in a variety of situations. The sets of attributes that students measure should expand as they progress: length, perimeter, area, volume, temperature, time, and angle measure. They should explore derived measurement such as speed, understand the role of units, and know how scale affects measurement. Students must see that all measurements are approximate, and that estimation skills play a significant role in life.

Goal 8 – Use algebraic and analytical methods to identify and describe patterns and relationships in data, solve problems, and predict results. Students must begin at an early age exploring patterns to build toward the understanding of functions. Their experience with numbers and patterns will form a foundation for symbols, properties, and algebraic expressions. Students must be able to use algebraic methods to construct and examine tables of values; to interpret the relationships expressed by patterns in these tables; to represent patterns in multiple ways including graphs and formulas; to reason about changes in quantities and the relationships involved in changes; and to find solutions to everyday problems using the tools of algebra and logic.

Goal 9 – Use geometric methods to analyze, categorize, and draw conclusions about points, lines, planes, and space. Students begin by learning about geometric shapes and how to analyze the characteristics and relationships by building and manipulating representations of two- and three-dimensional objects, as well as, perceiving objects from different perspectives. Geometric modeling and spatial reasoning offer ways for students to interpret and describe physical environments (e.g., maps, floor plans, art) and can be important tools in problem solving. Students must make connections between number patterns, geometry, and algebra (e.g., area representations of fractions, algebraic and geometric representations of number patterns). Students must make and explore conjectures about geometry and learn to reason carefully about geometric ideas. They must build an understanding across the grades, from informal to more formal thinking, developing the ideas of reasoning and proof. Technology plays an important role by facilitating many ways to explore conjectures.

Goal 10 – Collect, organize, and analyze data using statistical methods; predict results; and interpret uncertainty using concepts of probability. Students must be able to organize data; identify patterns in the data; and judge, using logic, the reasonableness of any claims and interpretations. Students must collect, display, and interpret data from a variety of sources (e.g., opinion polls, stock prices, tax rates, crime statistics, scientific studies, weather reports) to answer specific questions. They also must construct, analyze, and evaluate arguments that involve data, interpretation, and logical predictions. All students need to understand and apply the role probability plays in decision-making.

The ISBE Mathematics Expanded Performance Descriptors:

Intended Use and Interpretation

When the task force charged with writing the expanded performance descriptors for mathematics met and discussed the task, we were excited about the possibilities and the potential for helping teachers. In order for the materials to meet their potential, there are several key points the writers would like to bring to your attention before you start looking at this document and begin to implement changes in the classroom that reflect these expectations for students. What follows is important to the intended use and implementation of this document.

1) **Different goals receive different amounts of emphasis each year.** The Third International Mathematics and Science Study (TIMSS) data has shown us that the U.S. has traditionally tried to teach every area of mathematics every year, resulting in a curriculum that is "a mile wide and an inch deep". As a result, it has often been recommended that we teach fewer topics each year and teach them in greater depth. The NCTM *Principles and Standards for School Mathematics* (2000) are still grouped by grade bands, not individual grade levels, in part, because of this very concern. It also has a section in the beginning of chapter three dedicated to this topic, entitled “Growth across the Grades: Aiming for Focus and Coherence” (p. 30). The following is a short excerpt from that document.

> The Table of Standards and expectations in the appendix highlights the growth of expectations across the grades. It is not expected that every topic will be addressed each year. Rather, students will reach a certain depth of understanding of the concepts and acquire certain levels of fluency with the procedures by prescribed points in the curriculum, so further instruction can assume and build on this understanding and fluency.
>
> Even though each of these ten Standards applies to all grades, emphases will vary both within and between the grade bands. For instance, the emphasis on number is greatest in pre-kindergarten through grade two, and by grades nine through twelve, number receives less instructional attention. And the total time for mathematical instruction will be divided differently according to particular needs in each grade band—for example, in the middle grades, the majority of instructional time would address algebra and geometry. Figure 3.1 shows roughly how the Content Standards might receive different emphases across the grade bands.

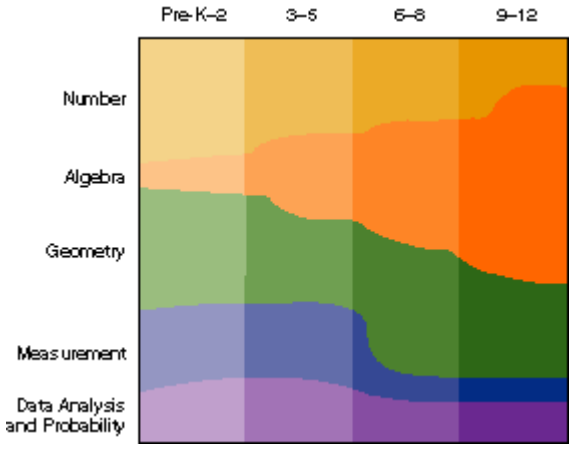


Fig. **3.1.** The Content Standards should receive different emphases across the grade bands (NCTM, 2000, p.30).

2) **The document is intended to be a developmental guide.** Stages are not intended to define grade levels. Several stages correspond to levels where ISAT will occur, and the performance descriptors match the "meets" standards at those levels. However, one stage below or above may not represent one grade level below or above. Instead, they represent the developmental stages of student learning, and show a progression through which students develop understanding of mathematical concepts. As a result, it is not enough for a teacher to look at a single stage and decide what content he or she will teach that year. He or she must look at a series of three stages to see the progression of understanding students should experience in order to move from one ISAT stage to the next. Stage C **does** correspond to the third grade ISAT expectations, Stage E to fifth grade, Stage H to eighth grade, Stage I to early high school, and Stage J to late high school. The other stages **are not** meant to explicitly correspond to the missing grades between.

3) **The document is a curriculum development tool.** The stages of development presented in this document can help a school district develop curriculum that will meet state standards. By using the stages corresponding to ISAT, curricula can move students through the stages of development without teaching every topic every year. For instance, when circle graphs are listed as a descriptor in Goal 10, Stage F, it means that the topic should be mastered before going on to further study of data displays and interpretation that will occur in Stage G and Stage H. There are multiple ways this could be handled. For example:
   - It is very possible that all three stages F, G, and H could be accomplished within a single year of study at seventh grade, if the seventh grade year were devoted to the study of three of four major topics, one being statistics.
   - Another possibility is combining circle graphs with the study of percent in grade six where a focus of the entire school year could be rational numbers, ratios, percents, and proportions. All mathematical content taught would then relate to these ideas, including the study and use of graphs, geometry (such as similar figures), or probability to help present fractions and ratios. So while many areas of mathematics are included, there are not separate units on geometry or statistics, but some of these ideas are incorporated into the main themes of the school year.

In this manner, the EPDs are used to help develop a curriculum rather than to prescribe a state-mandated curriculum of what should be taught at each grade.

4) **It is intended that each stage represent growth from the previous stage.** An idea is repeated in a stage only if new content or concepts are introduced with respect to that topic. For instance, once the concept of multi-digit addition and subtraction is placed in stage B, it is not repeated in stage C or above. It is assumed that students will expand on their knowledge and skills in the use of addition and subtraction, but it is not a new concept to be mastered. This is designed to remove isolated review from the curriculum. All review should be done in context of the new content being presented. For example, it is intended that there be no direct teaching of whole number computation beyond Stage D. By the time students reach Stage E, they should have mastered whole number computation. Similarly, all computation with rational numbers should be introduced and taught during stage E through stage H, and no computation with rational numbers should be directly taught after 8th grade. The writers realize that students who are below standards will need remedial work and extra help beyond the regular instructional time.

5) **The document uses appropriate mathematical terminology.** Occasionally, a reader may encounter an unfamiliar term. For example, in the early stages of geometry instruction the terms “slides”, “flips”, and “turns” are often used to describe translations, reflections, and rotations. It is our hope that a vocabulary shift will occur and the correct mathematical terms will be used. In order to assist the reader with terminology used in this document, a glossary has been written with specific mathematical terms, as well as verbs like "develop" to clarify what is meant by developing a concept or an idea.

6) **The document is based on the current form of the state learning goals and their benchmarks.** It is assumed that the state goals and benchmarks will undergo continual refinement, and this document will need to be revised as that happens. This document has also been carefully aligned with the NCTM *Principles and Standards for School Mathematics* as indicated by * and **. All descriptors are written to indicate the "meets" level of performance for each stage and are not intended to limit instruction. Curriculum should be fluid and change as needed. As such, this document should always be considered a work in progress.

Mathematics Performance Descriptors

6A Students who meet the standard can demonstrate knowledge and use of numbers and their many representations in a broad range of theoretical and practical settings. *(Representations)*

Stage A	Stage B	Stage C
1. Count with understanding, including skip counting by 2's, 5's, and 10's from zero. **	1. Count with understanding, including skip counting from any number by 2's and 10's.	1. Represent, order, and compare whole numbers to demonstrate an understanding of the base-ten number system.
2. Recognize 'how many' in sets of objects. **	2. Extend initial understanding of place value and the base-ten number system using multiple models. **	2. Recognize equivalent representations of whole numbers and generate them by composing and decomposing numbers (e.g., 123 = 100 + 20 + 3). **
3. Demonstrate the concept of odd and even using manipulatives.	3. Describe numeric relationships using comparison notation.	3. Judge the size of fractions using models, benchmarks, and equivalent forms. **
4. Develop initial understanding of place value and the base-ten number system using manipulatives. **	4. Use cardinal and ordinal numbers appropriately.	4. Represent, order, label, and compare familiar fractions.
5. Describe numeric relationships using appropriate vocabulary.	5. Recognize and explain the concept of odd and even numbers.	5. Recognize and generate equivalent forms of familiar fractions. **
6. Differentiate between cardinal and ordinal numbers in quantifying and ordering numbers.	6. Describe parts of a set using 1/2, 1/3, and 1/4.	6. Explore and discuss uses of decimals.
7. Connect number words and numerals to the quantities they represent. **	7. Represent, order, label, and compare unit fractions using concrete materials.	
8. Describe parts of a whole using 1/2, 1/3, and 1/4.		
9. Order concrete representations of unit fractions.		
Grade 1 (A-B) Grade 2 (A-B-C)	Grade 3 (B-C-D) Grade 4 (C-D-E)	Grade 5 (D-E-F)

* National Council of Teachers of Mathematics. *Principles and Standards for School Mathematics*. Reston, Va: National Council of Teachers of Mathematics, 2000.
** Adapted from: National Council of Teachers of Mathematics. *Principles and Standards for School Mathematics*. Reston, Va: National Council of Teachers of Mathematics, 2000.

Mathematics Performance Descriptors

6A Students who meet the standard can demonstrate knowledge and use of numbers and their many representations in a broad range of theoretical and practical settings. *(Representations)*

Stage D	Stage E	Stage F
1. Represent, order, and compare decimals to demonstrate understanding of the place-value structure in the base-ten number system. ** 2. Identify prime numbers through 100. 3. Recognize equivalent representations for decimals and generate them by composing and decomposing numbers (e.g., 0.15 = 0.1 + 0.05). 4. Represent fractions as parts of unit wholes, as parts of a set, as locations on a number line, and as divisions of whole numbers. ** 5. Explore numbers less than zero by extending a number line and through familiar applications. *	1. Place mixed numbers and decimals on a number line. 2. Show equivalent representations of a number by changing from one form to another form (e.g., standard form to expanded form, fraction to decimal, decimal to percent, improper fraction to mixed number). 3. Differentiate how fractions are used (part of a whole, part of a set, location on a number line, and division of a whole number). 4. Analyze how the size of the whole affects the size of the fraction (e.g., 1/2 of a large pizza is not the same as 1/2 of a small pizza). 5. Describe integers using familiar applications (e.g., a thermometer, above/below sea level).	1. Represent place values from units through billions using powers of ten. 2. Represent, order, compare, and graph integers. 3. Identify fractional pieces that have the same value but different shapes. 4. Compare and order fractions and decimals efficiently and find their approximate position on a number line. ** 5. Represent repeated factors using exponents.
Grade 1 (A-B) Grade 2 (A-B-C)	Grade 3 (B-C-D) Grade 4 (C-D-E)	Grade 5 (D-E-F)

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Mathematics Performance Descriptors

6B Students who meet the standard can investigate, represent and solve problems using number facts, operations, and their properties, algorithms, and relationships. *(Operations and properties)*

Stage A	Stage B	Stage C
1. Solve one-step addition and subtraction number sentences and word problems using concrete materials. 2. Construct number sentences to match word problems. 3. Demonstrate and describe the effects of adding and subtracting whole numbers using appropriate mathematical notation and vocabulary. ** 4. Explore and apply properties of addition and subtraction. 5. Compute using fact families.	1. Solve two-step addition and subtraction number sentences and word problems. 2. Demonstrate the relationship between addition and subtraction. 3. Explore multiplication and division through equal grouping and equal sharing of objects.** 4. Connect repeated addition to multiplication. 5. Demonstrate fluency with basic addition and subtraction facts. **	1. Show and use the relationship between multiplication and division. 2. Demonstrate and describe the effects of multiplying and dividing whole numbers using appropriate mathematical notation and vocabulary. 3. Explore, identify, and use relationships between and among properties of operations (e.g., commutativity applies to addition but not to subtraction). 4. Demonstrate fluency with basic multiplication and division facts. 5. Solve multiplication and division number sentences and word problems. 6. Apply knowledge of basic multiplication facts (factors 0-10) to related facts (e.g., 3 x 4 = 12, 30 x 4 = 120, 300 x 4 = 1200). 7. Select and use one of various algorithms to add and subtract.
Grade 1 (A-B) Grade 2 (A-B-C)	Grade 3 (B-C-D) Grade 4 (C-D-E)	Grade 5 (D-E-F)

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Mathematics Performance Descriptors

6B Students who meet the standard can investigate, represent and solve problems using number facts, operations, and their properties, algorithms, and relationships. *(Operations and properties)*

Stage D	Stage E	Stage F
1. Describe classes of numbers according to characteristics such as factors and multiples. **	1. Determine whether a number is prime or composite.	1. Write prime factorizations of numbers.
2. Solve addition or subtraction number sentences and word problems using fractions with like denominators.	2. Identify all the whole number factors of a composite number.	2. Determine the least common multiple and the greatest common factor of a set of numbers.
3. Solve multi-step number sentences and word problems using whole numbers and the four basic operations.	3. Explore and identify properties of square numbers.	3. Demonstrate the meaning of multiplication of fractions (e.g., 1/2 x 3 is 1/2 of a group of three objects).
4. Select and use one of various algorithms to multiply and divide.	4. Compute with 10, 100, 1000, and other powers of 10.	4. Simplify simple arithmetic expressions with rational numbers using the field properties and the order of operations.
	5. Explore and use divisibility rules.	5. Recognize and use the inverse relationships of addition and subtraction, multiplication and division to simplify computations and solve problems. **
	6. Solve number sentences and word problems using addition and subtraction of fractions with unlike denominators.	6. Solve multiplication number sentences and word problems with whole numbers and familiar fractions.
	7. Solve number sentences and word problems using addition and subtraction of decimals.	
Grade 1 (A-B) Grade 2 (A-B-C) Grade 3 (B-C-D) Grade 4 (C-D-E) Grade 5 (D-E-F)		

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Mathematics Performance Descriptors

6C Students who meet the standard can compute and estimate using mental mathematics, paper-and-pencil methods, calculators, and computers. *(Choice of method)*

Stage A	Stage B	Stage C
1. Develop and use strategies for whole number computations with a focus on addition and subtraction. *	1. Explain and use mental math strategies to solve simple addition and subtraction problems.	1. Develop and use strategies (i.e. rounding) to estimate the results of whole-number computations and to judge the reasonableness of such results. **
2. Use mental math counting strategies.	2. Estimate sums and differences of one- or two-digit numbers.	2. Select appropriate methods and tools for computing with whole numbers from mental computation, estimation, calculators, and paper/pencil according to the context and nature of the computation and use of the selected method or tool. *
3. Describe reasonable and unreasonable sums and differences.	3. Analyze situations to determine whether exact numbers or estimates are appropriate.	**3.** Determine whether exact answers or estimates are appropriate for solutions to problems.
4. Utilize a calculator for counting patterns.	4. Utilize a calculator to solve addition and subtraction problems.	
Grade 1 (A-B) Grade 2 (A-B-C)	Grade 3 (B-C-D) Grade 4 (C-D-E)	Grade 5 (D-E-F)

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Mathematics Performance Descriptors

6C Students who meet the standard can compute and estimate using mental mathematics, paper-and-pencil methods, calculators, and computers. *(Choice of method)*

Stage D	Stage E	Stage F
1. Develop and use strategies (e.g., compatible numbers, front-end estimation) to estimate the results of whole-number computations and to judge the reasonableness of such results. ** 2. Estimate the sum or difference of a number sentence containing decimals using a variety of strategies.	1. Develop and use strategies to estimate computations involving familiar fractions and decimals in situations relevant to students' experience * (e.g., double a recipe with 3/8 cup sugar, will more than a cup of sugar be needed). 2. Evaluate estimates to judge their reasonableness and degree of accuracy. 3. Select and use appropriate operation(s) and tool(s) (e.g., mental math, pencil-and-paper, estimation, calculator, computer) to perform calculations on whole numbers, fractions, and decimals according to the context and nature of the computation. ** 4. Determine and justify whether exact answers or estimates are appropriate.	1. Select and use appropriate operations, methods, and tools to compute or estimate using whole numbers with natural number exponents. ** 2. Analyze algorithms for computing with whole numbers, familiar fractions, and decimals and develop fluency in their use. **
Grade 1 (A-B) Grade 2 (A-B-C)	Grade 3 (B-C-D) Grade 4 (C-D-E)	Grade 5 (D-E-F)

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Mathematics Performance Descriptors

6D Students who meet the standard can solve problems using comparison of quantities, ratios, proportions, and percents.

Stage A	Stage B	Stage C
1. Compare two or more sets, using manipulatives, to solve problems.	1. Compare unit fractions, using manipulatives, to solve problems.	1. Describe the relationship between two sets using ">", "<", and "=", "≠".
Grade 1 (A-B) Grade 2 (A-B-C) Grade 3 (B-C-D) Grade 4 (C-D-E) Grade 5 (D-E-F)		

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Mathematics Performance Descriptors

6D Students who meet the standard can solve problems using comparison of quantities, ratios, proportions, and percents.

Stage D	Stage E	Stage F
1. Determine 50% and 100% of a given group in context.	1. Identify and express ratios using appropriate notation (i.e., a/b, a to b, a:b). 2. Model the concept of percent using manipulatives or drawings.	1. Solve number sentences and word problems using percents. 2. Demonstrate and explain the meaning of percents, including greater than 100 and less than 1. ** 3. Create and explain a pattern that shows a constant ratio. 4. Analyze situations to determine whether ratios are appropriate to solve problems. 5. Determine equivalent ratios.
Grade 1 (A-B) Grade 2 (A-B-C) Grade 3 (B-C-D) Grade 4 (C-D-E) Grade 5 (D-E-F)		

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Mathematics Performance Descriptors

7A Students who meet the standard can measure and compare quantities using appropriate units, instruments, and methods. *(Performance and conversion of measurements)*

Stage A	Stage B	Stage C
1. Determine the attributes of an object that are measurable (e.g., length and weight are measurable; color and texture are not). 2. Compare and order objects according to measurable attributes. ** 3. Measure objects using non-standard units. 4. Explore and describe chronological events (e.g., calendars, timelines, seasons). 5. Identify units of money and the value of each. 6. Count like sets of coins.	1. Identify the type of measure (e.g., weight, height, volume, temperature) for each measurable attribute. 2. Measure objects using standard units. 3. Order events chronologically. 4. Tell time using an analog clock. 5. Describe relationships within units of time, money, and length (e.g., 12 inches in a foot). 6. Count, compare, and order sets of unlike coins. 7. Show equivalent amounts of money. 8. Explore and explain making change using manipulatives.	1. Explain the need for using standard units for measuring. ** 2. Measure objects using standard units in the U.S. customary and metric systems. ** 3. Perform simple unit conversions within a system of measurement (e.g., three feet is the same as a yard). ** 4. Describe multiple measurable attributes (e.g., length, mass/weight, time, temperature, area, volume, capacity) of a single object. 5. Show and explain perimeter of an object by measuring and adding its linear units. 6. Show and explain the area of an object by counting square units.
Grade 1 (A-B) Grade 2 (A-B-C)	Grade 3 (B-C-D) Grade 4 (C-D-E)	Grade 5 (D-E-F)

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Mathematics Performance Descriptors

7A Students who meet the standard can measure and compare quantities using appropriate units, instruments, and methods. *(Performance and conversion of measurements)*

Stage D	Stage E	Stage F
1. Measure angles using a protractor or angle ruler. 2. Measure with a greater degree of accuracy. 3. Convert U.S. customary measurements into larger or smaller units with the help of conversion charts. 4. Convert linear metric measurements into larger or smaller units with the help of a conversion chart.	1. Convert U.S. customary and metric measurements into larger or smaller units. 2. Draw an angle of any given measure using a protractor or angle ruler.	1. Investigate the history of the U.S. customary and metric systems of measurement. 2. Measure, with a greater degree of accuracy, any angle using a protractor or angle ruler.
Grade 1 (A-B) Grade 2 (A-B-C) Grade 3 (B-C-D) Grade 4 (C-D-E) Grade 5 (D-E-F)		

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Mathematics Performance Descriptors

7B Students who meet the standard can estimate measurements and determine acceptable levels of accuracy. *(Estimation)*

Stage A	Stage B	Stage C
1. Estimate nonstandard measurements of length, weight, and capacity.	1. Estimate elapsed time for a given task. 2. Estimate standard measurements of length, weight, and capacity. 3. Estimate the amount of money needed to make purchases.	1. Develop and use common referents for linear measures to make comparisons and estimates. **2.** Estimate perimeter of simple polygons.
Grade 1 (A-B) Grade 2 (A-B-C)	Grade 3 (B-C-D) Grade 4 (C-D-E)	Grade 5 (D-E-F)

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Mathematics Performance Descriptors

7B Students who meet the standard can estimate measurements and determine acceptable levels of accuracy. *(Estimation)*

Stage D	Stage E	Stage F
1. Develop and discuss strategies for estimating the perimeters, areas, and volumes of regular and non-regular shapes. ** 2. Develop and use common referents for volume, weight/mass, capacity, area, and angle measures to make comparisons and estimates.	1. Explain that all measurements are approximations. 2. Describe how precision is affected by choice of units. 3. Estimate the perimeter, area, and/or volume of regular and irregular shapes and objects.	1. Estimate distance, weight, temperature, and elapsed time using reasonable units and with acceptable levels of accuracy.
Grade 1 (A-B) Grade 2 (A-B-C) Grade 3 (B-C-D) Grade 4 (C-D-E) Grade 5 (D-E-F)		

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Mathematics Performance Descriptors

7C Students who meet the standard can select and use appropriate technology, instruments, and formulas to solve problems, interpret results, and communicate findings. *(Progression from selection of appropriate tools and methods to application of measurements to solve problems)*

Stage A	Stage B	Stage C
1. Select appropriate nonstandard measurement units to measure length, weight, and capacity (e.g., number of handfuls of cubes to fill a container).	1. Select an appropriate unit and tool for measurement. ** 2. Explore and describe perimeter and area of real objects. 3. Solve problems using money and time.	1. Select and apply appropriate standard units and tools to measure length, area, volume, weight, time, and temperature. * 2. Determine elapsed time between events. 3. Solve problems using perimeter and area of simple polygons. 4. Make change from a given amount using bills and coins.
Grade 1 (A-B) Grade 2 (A-B-C)	Grade 3 (B-C-D) Grade 4 (C-D-E)	Grade 5 (D-E-F)

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Mathematics Performance Descriptors

7C Students who meet the standard can select and use appropriate technology, instruments, and formulas to solve problems, interpret results, and communicate findings. *(Progression from selection of appropriate tools and methods to application of measurements to solve problems)*

Stage D	Stage E	Stage F
1. Select and apply appropriate standard units and tools to measure the size of angles. ** 2. Determine the volume of a cube or rectangular prism using concrete materials. 3. Create an accurate representation of a polygon with a given perimeter or area.	1. Select appropriate tools to measure, draw, or construct figures. 2. Develop and discuss strategies for determining area and perimeter of irregular shapes. 3. Develop and use formulas to determine the area of squares, rectangles, and right triangles. 4. Read and interpret a scale on a map or a scale drawing using the idea of a constant ratio (e.g., 1" represents 1 mile), and use it to answer questions about actual measurement. 5. Explain the meaning of a measurement answer in context.	1. Select and justify an appropriate formula to find the area of triangles, parallelograms, and trapezoids. ** 2. Select an appropriate formula or strategy to find the surface area and volume of rectangular and triangular prisms. ** 3. Develop and use formulas for determining the area of triangles, parallelograms, and trapezoids. 4. Develop and use the formula for determining the volume of a rectangular and triangular prism. 5. Calculate the surface area of a cube, rectangular prism, and triangular prism. 6. Develop and use formulas for determining the circumference and area of circles.
Grade 1 (A-B) Grade 2 (A-B-C)	Grade 3 (B-C-D) Grade 4 (C-D-E)	Grade 5 (D-E-F)

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Mathematics Performance Descriptors

8A Students who meet the standard can describe numerical relationships using variables and patterns. *(Representations and algebraic manipulations)*

Stage A	Stage B	Stage C
1. Describe common and uncommon attributes (all, some, none) in a set. 2. Recognize, describe, and extend patterns such as sequences of sounds, motions, shapes, or simple numeric patterns, and translate from one representation to another (e.g., red-blue-red-blue translates to snap-clap-snap-clap). ** 3. Describe given patterns using letters. 4. Analyze repeating patterns. **	1. Sort, classify, and order objects by multiple properties. ** 2. Create rules for multiple sortings in a single set. 3. Recognize, describe, and extend geometric and numeric patterns. 4. Create patterns concretely and numerically to match a given letter description (e.g., AAB) and make predictions. 5. Extend numeric patterns involving addition and/or subtraction (e.g., 1, 3, 5, … what are the next two terms?). 6. Change patterns by manipulation of concrete materials. 7. Describe missing units in a pattern. 8. Analyze growing patterns.	1. Extend geometric and simple numeric patterns using concrete objects or paper and pencil. 2. Demonstrate how to create a pattern given a set of directions. 3. Identify errors in a given pattern. 4. Represent the idea of a variable as an unknown quantity using a letter or a symbol in a numerical sentence. ** 5. Express mathematical relationships using equations.
Grade 1 (A-B) Grade 2 (A-B-C)	Grade 3 (B-C-D) Grade 4 (C-D-E)	Grade 5 (D-E-F)

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Mathematics Performance Descriptors

8A Students who meet the standard can describe numerical relationships using variables and patterns. *(Representations and algebraic manipulations)*

Stage D	Stage E	Stage F
1. Identify a number pattern, both increasing and decreasing, and extend the number sequence. 2. Determine the missing number(s) in a complex repeating pattern. 3. Construct and solve simple number sentences using a symbol for a variable. 4. Make generalizations given a specific pattern. 5. Create, describe, and extend patterns. 6. Describe a pattern with one operation, verbally and symbolically, given a table of input/output numbers.	1. Describe, extend, and make generalizations about given geometric and numeric patterns. ** 2. Describe a pattern, with at least two operations, verbally and symbolically, given a table of input/output numbers. 3. Demonstrate equality of two expressions with variables (e.g., 28 + 35 = 35 + *n*). 4. Describe situations involving inverse relationships (e.g., the more people, the fewer cookies per person).	1. Investigate, extend, and describe arithmetic and geometric sequences of numbers whether presented in numeric or pictorial form. ** 2. Evaluate algebraic expressions for given values. 3. Express properties of numbers and operations using variables (e.g., the commutative property is m + n = n + m). 4. Simplify algebraic expressions involving like terms.
Grade 1 (A-B) Grade 2 (A-B-C)	Grade 3 (B-C-D) Grade 4 (C-D-E)	Grade 5 (D-E-F)

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Mathematics Performance Descriptors

8B Students who meet the standard can interpret and describe numerical relationships using tables, graphs, and symbols. *(Connections of representations including the rate of change)*

Stage A	Stage B	Stage C
1. Describe and compare qualitative change, (e.g., student grows taller). **	1. Describe and compare quantitative change (e.g., student grows two inches in one year). **	**1.** Represent and analyze simple patterns and operations using words, tables, and graphs. ** 2. Describe situations with constant rates of change using words, tables, and graphs (e.g., walking at a constant rate of speed).
Grade 1 (A-B) Grade 2 (A-B-C) Grade 3 (B-C-D) Grade 4 (C-D-E) Grade 5 (D-E-F)		

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Mathematics Performance Descriptors

8B Students who meet the standard can interpret and describe numerical relationships using tables, graphs, and symbols. *(Connections of representations including the rate of change)*

Stage D	Stage E	Stage F
1. Create a table that describes a function rule for a single operation. 2. Demonstrate, in simple situations, how a change in one quantity results in a change in another quantity (e.g., increase the measure of the side of a square and the perimeter increases). 3. Identify situations with varying rates of change using words, tables, and graphs (e.g., growth of a plant). **	1. Model problem situations with objects and equations to draw conclusions. ** 2. Represent and analyze patterns and functions using words, tables, and graphs. * 3. Demonstrate how the change in one quantity affects the other in a functional relationship involving whole numbers and unit fractions. 4. Identify, describe, and compare situations with constant and varying rates of change using words, tables, and graphs (e.g., two quantities that vary together are the length of the side of a square and its area). **	1. Graph simple inequalities on a number line. 2. Create a table of values that satisfy a simple linear equation and plot the points on the Cartesian plane. 3. Describe verbally, symbolically, and graphically, a simple relationship presented by a set of ordered pairs of numbers.
Grade 1 (A-B) Grade 2 (A-B-C)	Grade 3 (B-C-D) Grade 4 (C-D-E)	Grade 5 (D-E-F)

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Mathematics Performance Descriptors

8C Students who meet the standard can solve problems using systems of numbers and their properties. *(Problem solving; number systems, systems of equations, inequalities, algebraic functions)*

Stage A	Stage B	Stage C
1. Solve simple number sentences with variables (e.g., missing addend problems).	1. Solve word problems involving unknown quantities. 2. Apply the relationship of addition and subtraction families to solve for an unknown quantity.	1. Apply the relationship of multiplication and division fact families to solve for an unknown quantity.
Grade 1 (A-B) Grade 2 (A-B-C) Grade 3 (B-C-D) Grade 4 (C-D-E) Grade 5 (D-E-F)		

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Mathematics Performance Descriptors

8C Students who meet the standard can solve problems using systems of numbers and their properties. *(Problem solving; number systems, systems of equations, inequalities, algebraic functions)*

Stage D	Stage E	Stage F
1. Solve problems with whole numbers using appropriate field properties.	1. Solve problems with whole numbers using order of operations, equality properties, and appropriate field properties.	1. Identify and explain incorrect uses of the commutative, associative, and distributive properties. 2. Identify and provide examples of the identity property of addition and multiplication. 3. Identify and provide examples of inverse operations. 4. Explain why division by zero is undefined.
Grade 1 (A-B) Grade 2 (A-B-C) Grade 3 (B-C-D) Grade 4 (C-D-E) Grade 5 (D-E-F)		

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Mathematics Performance Descriptors

8D Students who meet the standard can use algebraic concepts and procedures to represent and solve problems. *(Connection of 8A, 8B, and 8C to solve problems)*

Stage A	Stage B	Stage C
1. Solve real life word problems using patterns.	1. Solve problems and justify solutions using patterns.	1. Demonstrate how to select and use an appropriate operation to solve problems involving patterns (e.g., save one penny on day 1, double that amount each day for 10 days). 2. Solve one-step linear equations using concrete materials.
Grade 1 (A-B) Grade 2 (A-B-C) Grade 3 (B-C-D) Grade 4 (C-D-E) Grade 5 (D-E-F)		

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Mathematics Performance Descriptors

8D Students who meet the standard can use algebraic concepts and procedures to represent and solve problems. *(Connection of 8A, 8B, and 8C to solve problems)*

Stage D	Stage E	Stage F
1. Solve one-step linear equations with one missing value in isolation and in problem solving situations.	1. Create and solve linear equations involving whole numbers using a variety of methods (e.g., guess and check, bean stick counters).	1. Create, model, and solve algebraic equations using concrete materials. 2. Solve linear equations, including direct variation, with whole number coefficients and solutions using algebraic or graphical representations.
Grade 1 (A-B) Grade 2 (A-B-C) Grade 3 (B-C-D) Grade 4 (C-D-E) Grade 5 (D-E-F)		

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Mathematics Performance Descriptors

9A Students who meet the standard can demonstrate and apply geometric concepts involving points, lines, planes, and space. *(Properties of single figures, coordinate geometry and constructions)*

Stage A	Stage B	Stage C
1. Identify two- and three-dimensional shapes. ** 2. Model two-dimensional geometric shapes by drawing or building. ** 3. Describe and interpret relative positions in space and apply concepts of relative position (e.g., above/below). ** 4. Recognize and describe shapes that have line symmetry. ** **5.** Identify geometric shapes and structures in the environment. ** **6.** Explore the effects of translations (slides), reflections (flips), and rotations (turns) with concrete objects.	1. Investigate and predict the results of putting together and taking apart two- and three-dimensional shapes (e.g., put two triangles together to make a quadrilateral). ** 2. Describe and interpret direction and distance in navigating space, and apply concepts of direction and distance (e.g., nearer/farther). ** 3. Perform translations (slides), reflections (flips), and rotations (turns) with concrete objects. 4. Create and complete shapes that have line symmetry.	1. Specify locations using a coordinate system. ** 2. Predict and describe the results of translations, rotations, and reflections of two-dimensional shapes. **3.** Identify, draw, and build polygons.
Grade 1 (A-B) Grade 2 (A-B-C)	Grade 3 (B-C-D) Grade 4 (C-D-E)	Grade 5 (D-E-F)

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Mathematics Performance Descriptors

9A Students who meet the standard can demonstrate and apply geometric concepts involving points, lines, planes, and space. *(Properties of single figures, coordinate geometry and constructions)*

Stage D	Stage E	Stage F
1. Identify, draw, and label lines, line segments, rays, parallel lines, intersecting lines, perpendicular lines, acute angles, obtuse angles, right angles, and acute, obtuse, right, scalene, isosceles, and equilateral triangles. 2. Identify, draw, and build regular, irregular, convex, and concave polygons. 3. Read and plot ordered pairs of numbers in the positive quadrant of the Cartesian plane. 4. Describe paths and movement using coordinate systems. 5. Differentiate between polygons and non-polygons. 6. Identify and label radius, diameter, chord, and circumference of a circle. 7. Explore and describe rotational symmetry of two- and three-dimensional shapes. ** **8.** Construct a circle with a specified radius or diameter using a compass.	1. Identify, compare, and analyze attributes of two- and three-dimensional shapes and develop vocabulary to describe the attributes. * 2. Classify two- or three-dimensional shapes according to their properties (e.g., regular and irregular, concave and convex, types of quadrilaterals, pyramids, and prisms). ** 3. Investigate and describe the results of subdividing and combining shapes. ** 4. Describe paths using coordinate systems. ** 5. Determine the distance between points along horizontal and vertical lines of a coordinate system. ** 6. Identify and justify rotational symmetry in two- and three-dimensional shapes. ** 7. Identify and describe how geometric figures are used in practical settings (e.g., construction, art, advertising, architecture). 8. Identify, sketch, and build two- and three-dimensional shapes given attribute clues. 9. Copy a line segment or an angle using a straightedge and a compass. 10. Construct a perpendicular bisector of a line segment.	1. Plot and read ordered pairs of numbers in all four quadrants. 2. Describe sizes, positions, and orientations of shapes under transformations, including dilations. 3. Perform simple constructions (e.g., equal segments, angle and segment bisectors, or perpendicular lines, inscribing a hexagon in a circle) with a compass and straightedge or a mira. 4. Determine and describe the relationship between pi, the diameter, the radius, and the circumference of a circle. **5.** Determine unknown angle measures using angle relationships and properties of a triangle or a quadrilateral.
Grade 1 (A-B) Grade 2 (A-B-C)	Grade 3 (B-C-D) Grade 4 (C-D-E)	Grade 5 (D-E-F)

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Mathematics Performance Descriptors

9B Students who meet the standard can identify, describe, classify and compare relationships using points, lines, planes, and solids. *(Connections between and among multiple geometric figures)*

Stage A	Stage B	Stage C
1. Identify objects that are the same shape. **2.** Compare and sort two- and three-dimensional objects.	1. Identify objects that are congruent **2.** Compare and contrast attributes of two- and three-dimensional objects using appropriate vocabulary.	1. Decompose a three-dimensional object into two-dimensional components. 2. Describe the difference between congruence and similarity. ** 3. Describe a motion or a series of motions that will show that two shapes are congruent. * 4. Identify and build a three-dimensional object from two-dimensional representations of that object. * 5. Apply geometric ideas and relationships to problems that arise in the classroom or in everyday life. ** 6. Apply geometric ideas and relationships to other disciplines. **
Grade 1 (A-B) Grade 2 (A-B-C) Grade 3 (B-C-D) Grade 4 (C-D-E) Grade 5 (D-E-F)		

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Mathematics Performance Descriptors

9B Students who meet the standard can identify, describe, classify and compare relationships using points, lines, planes, and solids. *(Connections between and among multiple geometric figures)*

Stage D	Stage E	Stage F
1. Determine congruence and similarity of given shapes. ** 2. Explore polyhedra using concrete models.	1. Demonstrate congruence of plane figures using transformations (translation, rotation, reflection). 2. Determine if two polygons are congruent using measures of angles and sides. 3. Match a front, right side, and top view drawing with a three-dimensional model built with cubes. **4.** Identify and describe the five regular polyhedra. **5.** Create regular and semi-regular tessellations using pattern blocks, other manipulatives, or technology.	1. Determine the relationships between the number of vertices or sides in a polygon, the number of diagonals, and the sum of its angles. 2. Solve problems that involve vertical, complementary, and supplementary angles. 3. Analyze quadrilaterals for defining characteristics. 4. Create a three-dimensional object from any two-dimensional representation of the object, including multiple views, nets, or technological representations.
Grade 1 (A-B) Grade 2 (A-B-C) Grade 3 (B-C-D) Grade 4 (C-D-E) Grade 5 (D-E-F)		

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Mathematics Performance Descriptors

9C Students who meet the standard can construct convincing arguments and proofs to solve problems. *(Justifications of conjectures and conclusions)*

Stage A	Stage B	Stage C
1. Recognize and explain a geometric pattern.	1. Justify an extension of a pattern.	1. Make and test conjectures about mathematical properties and relationships and justify the conclusions. **
Grade 1 (A-B) Grade 2 (A-B-C) Grade 3 (B-C-D) Grade 4 (C-D-E) Grade 5 (D-E-F)		

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Mathematics Performance Descriptors

9C Students who meet the standard can construct convincing arguments and proofs to solve problems. *(Justifications of conjectures and conclusions)*

Stage D	Stage E	Stage F
1. Make and test conjectures about mathematical properties and relationships and justify the conclusions. **	1. Make and test conjectures about mathematical properties and relationships and develop logical arguments to justify conclusions. ** 2. Make and test conjectures about the results of subdividing and combining shapes. ** **9D is Not Applicable for Stages A – F**	1. Make, test, and justify conjectures about various quadrilateral and triangle relationships, including the triangle inequality. 2. Justify the relationship between vertical angles. 3. Justify that the sum of the angles of a triangle is 180 degrees.
Grade 1 (A-B) Grade 2 (A-B-C) Grade 3 (B-C-D) Grade 4 (C-D-E) Grade 5 (D-E-F)		

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Mathematics Performance Descriptors

10A Students who meet the standard can organize, describe and make predictions from existing data. *(Data analysis)*

Stage A	Stage B	Stage C
1. Organize, describe, and label simple data displays such as pictographs, tallies, tables, and bar graphs. 2. Compare numerical information derived from tables and graphs.	1. Organize and interpret simple data displays such as pictographs, tallies, tables, and bar graphs. 2. Make predictions from simple data.	1. Organize, describe, and make predictions from existing data. * 2. Represent data using tables and graphs such as tallies and bar graphs. 3. Describe the important features of a set of data displayed by a graph. 4. Determine the median of data on a graph.
Grade 1 (A-B) Grade 2 (A-B-C) Grade 3 (B-C-D) Grade 4 (C-D-E) Grade 5 (D-E-F)		

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Mathematics Performance Descriptors

10A Students who meet the standard can organize, describe and make predictions from existing data. *(Data analysis)*

Stage D	Stage E	Stage F
1. Represent data using tables and graphs such as line plots and line graphs. ** 2. Describe the shape and important features of a set of data and compare related data sets. ** 3. Arrange given data in order, least to greatest or greatest to least, and determine minimum value, maximum value, range, mode, and median for an odd number of data points. 4. Compare different representations of the same data and evaluate how well each representation shows important aspects of the data. * 5. Propose and justify conclusions and predictions that are based on data. **	1. Represent given data using double bar graphs, double line graphs, and stem and leaf plots with and without technology. 2. Select an appropriate graph format to display given data. 3. Read, interpret, infer, predict, draw conclusions, and evaluate data from any graph. 4. Determine mean, median, mode, minimum value, maximum value, and range, and discuss what each does to help interpret a given set of data.	1. Construct, read, interpret, infer, predict, draw conclusions, and evaluate data from various displays, including circle graphs. ** 2. Recognize and explain misleading displays of data due to inappropriate intervals on a scale.
Grade 1 (A-B) Grade 2 (A-B-C)	Grade 3 (B-C-D) Grade 4 (C-D-E)	Grade 5 (D-E-F)

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Mathematics Performance Descriptors

10B Students who meet the standard can formulate questions, design data collection methods, gather and analyze data and communicate findings. *(Data Collection)*

Stage A	Stage B	Stage C
1. Gather data to answer a simple question.	1. Gather data by creating and using interview questions.	1. Create and administer a survey considering which questions will be asked and how the answers will be recorded. 2. Propose a follow-up survey to investigate questions that arise from the initial survey.
Grade 1 (A-B) Grade 2 (A-B-C) Grade 3 (B-C-D) Grade 4 (C-D-E) Grade 5 (D-E-F)		

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Mathematics Performance Descriptors

10B Students who meet the standard can formulate questions, design data collection methods, gather and analyze data and communicate findings. *(Data Collection)*

Stage D	Stage E	Stage F
1. Collect data using observations and experiments. ** 2. Propose a further investigation to verify or refute a prediction. **	1. Design investigations to address a question and consider how data-collection methods affect the nature of a data set. 2. Propose and justify conclusions and predictions that are based on data, and design studies to further investigate the conclusions or predictions. *	1. Gather data by conducting simple simulations. 2. Collect data over time with or without technology.
Grade 1 (A-B) Grade 2 (A-B-C) Grade 3 (B-C-D) Grade 4 (C-D-E) Grade 5 (D-E-F)		

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** Adapted from: National Council of Teachers of Mathematics. *Principles and Standards for School Mathematics*. Reston, Va: National Council of Teachers of Mathematics, 2000.

Mathematics Performance Descriptors

10C Students who meet the standard can determine, describe and apply the probabilities of events. *(Probability including counting techniques)*

Stage A	Stage B	Stage C
1. Identify possible and impossible results of probability events using concrete materials. 2. Determine all possible outcomes of a given situation.	1. Identify and discuss likely, unlikely, and impossible probability events. 2. Communicate and display results of probability events in order to make predictions of future events.	**1.** Describe events as likely or unlikely and discuss the degree of likelihood using such words as certain, equally likely, and impossible. * **2.** Explain probability as a fractional part of a group to the whole group (e.g., A tossed coin can land on heads or tails; therefore, it should land on heads 1/2 of the time.) **3.** Make predictions based on the results received from a probability experiment. **4.** Create and perform a probability experiment (e.g., a penny is flipped 100 times) and record the results. **5.** Understand that the measure of the likelihood of an event can be represented by a number from zero to one, inclusive. **
Grade 1 (A-B) Grade 2 (A-B-C)	Grade 3 (B-C-D) Grade 4 (C-D-E)	Grade 5 (D-E-F)

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Mathematics Performance Descriptors

10C Students who meet the standard can determine, describe and apply the probabilities of events. *(Probability including counting techniques)*

Stage D	Stage E	Stage F
1. List all possible outcomes of a single event and tell whether an outcome is certain, impossible, likely, or unlikely. 2. Describe the probability of an event using terminology such as "5 chances out of 8."	1. List all possible outcomes of compound, independent events (e.g., toss a coin and spin a spinner). 2. Assign a value of zero to probabilities that are impossible and a value of one to probabilities that are certain. 3. Express simple probabilities as a fraction between zero and one. 4. Predict the probability of outcomes of simple experiments and test the predictions. *	1. Record probabilities as fractions, decimals, or percents. 2. Demonstrate that the sum of all probabilities equals one. 3. Determine empirical probabilities from a set of data provided. 4. Set up a simulation to model the probability of a single event. 5. Discuss the effect of sample size on the empirical probability compared to the theoretical probability. 6. List outcomes by a variety of methods (e.g., tree diagram). 7. Determine theoretical probabilities of simple events.
Grade 1 (A-B) Grade 2 (A-B-C)	Grade 3 (B-C-D) Grade 4 (C-D-E)	Grade 5 (D-E-F)

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GLOSSARY

ACCEPTABLE LEVELS OF ACCURACY - the precision determined by the situation or the given numbers, students should help develop what is acceptable according to the situation.

ADDITIVE IDENTITY PROPERTY - the sum of any number and zero is the original number; zero is the identity element of addition.

ANALYZE - to break down into parts and explain or demonstrate the logic of a situation or a process.

ANGLE RULER - a hinged ruler with a protractor attached for reading the measure of an angle in degrees.

ASSOCIATIVE PROPERTY - property about grouping of numbers; of addition, the formula $(a + b) + c = a + (b + c)$.

ASYMPTOTE - a straight line always approaching but never intersecting a curve.

BOX PLOT - a representation of data above a numbered scale where the "box" encloses all data between the median of the lower half (quartile 1) and the median of the upper half (quartile 3), with a vertical line inside the box to indicate the median of the data; a dot represents each of the high and low values of the data, and a horizontal line called a whisker connects each dot to the box.

CALCULATOR NOTATION - the symbols used by a calculator for scientific notation.

CAPACITY - the amount of liquid that can fill an object.

CARTESIAN PLANE - a rectangular coordinate system consisting of a horizontal number line (x-axis) and a vertical number line (y-axis), intersecting at the origin (zero on each number line).

CHORD - a straight line joining any two points on an arc, curve or circle.

CLASS OF FUNCTIONS - family of functions such as linear, quadratic, power (polynomial), exponential, or logarithmic.

CLASSES OF NUMBERS - family of numbers or number systems such as natural, integer, rational, irrational, real, or complex.

COMMUTATIVE PROPERTIES - properties about order - of addition, $a + b = b + a$; of multiplication, $a \times b = b \times a$.

COMPATIBLE NUMBERS - numbers that can be easily manipulated and operated on mentally.

COMPLEX NUMBERS - numbers of the form a + bi, where a and b are real numbers and i = $\sqrt{-1}$ -1.

COMPOSE NUMBERS - put a set of numbers together to form a new number using addition or multiplication.

COMPOSITION - see "compose numbers".

COMPOUND EVENTS - two or more events in a probability situation such as flipping a coin and spinning a spinner.

CONCRETE MATERIALS - objects to be manipulated (e.g., pattern blocks, snap cubes, geoboards, tangrams, color tiles, base ten blocks).

CONIC SECTION - figures represented by cross-sections of a cone or two cones placed apex to apex - parabola, circle, ellipse, hyperbola.

CONSTANT RATE OF CHANGE - set of data or table of values in which the amount of the dependent variable changes by a constant (fixed) value as the value of the independent variable changes by a constant value.

CONSTANT RATIO - see constant rate of change.

CONSTRUCT - create a figure using only a straight edge and compass.

CONTRACTION - the opposite of dilation, a figure resulting from multiplying all dimensions of a given figure by a number between zero and one.

CONVERSION FACTOR - relationship between two units from different systems of measurement used to convert from one system to the other (e.g., 2.54 centimeters corresponds to 1 inch).

COORDINATE SYSTEM - set of ordered pairs used to locate an object or point on the two-dimensional plane.

COUNTING TECHNIQUES - a variety of methods used to determine the total possible outcomes, typically in a probability situation, including the multiplication principle, trees and lists.

DECOMPOSE NUMBERS - break up numbers into addends or factors.

DEPENDENT VARIABLE - a variable whose value depends on the value of another variable in combination with or without other numbers.

DERIVED MEASUREMENT - a measurement that is a combination of two other measurements such as speed in miles per hour.

DESCRIBE - to explain orally or in writing.

DEVELOP – to be involved in reasoning, exploration, conjecturing, using manipulatives or sketches to gain understanding of concepts or relationships.

DEVELOP FLUENCY - to become skillful in working with numbers, both in accuracy and speed.

DILATIONS - a figure resulting from multiplying all dimensions of a given figure by a number greater than one.

DIRECT VARIATION - a variation of the type y = kx; the graph is a straight line through the origin.

DISTRIBUTIVE PROPERTY - the property which relates multiplication and addition; the formula, a(b + c) = a x b + a x c.

DOUBLE LINE GRAPHS - graphs in which two sets of data are graphed at the same time, connecting each set with line segments.

DRAW - to create a figure using numbered scales on tools such as rulers and protractors.

ERROR TOLERANCE - the value allowable above and below a number or its approximation.

EXACT NUMBER - a numerical result that has not been rounded or estimated.

EXPLICIT FORM - a formula for any term of a sequence given the number of the term.

EXPONENTIAL FUNCTION - a function of the form $y = a(b^x)$.

EXPRESSION - combination of numerals or numerals and variables that indicate a finite number of operations, not an equation.

EXTRAPOLATE - to use given information to predict values beyond the set of given values using either a formula or a reasonable estimate.

FAMILIAR FRACTIONS - commonly used fractions such as halves, thirds, fourths, fifths, sixths, eighths and tenths.

FIELD PROPERTIES - closure for addition and multiplication, commutative for addition and multiplication, associative for addition and multiplication, identity for addition and multiplication, inverse for addition and multiplication, distributive for multiplication over addition.

FLEXIBLY - usually applied to computation, where students should be able to mentally manipulate numbers and components of numbers to create a solution to a problem.

FUNCTION RULE - the set of operations that describes the process that takes the independent variable and transforms it into the dependent variable in a consistent way.

GREATEST COMMON FACTOR - the greatest number that is a factor of each of the given numbers.

GROWING PATTERN – a pattern where the number of objects in the pattern increases from term to term.
GROWTH FORMULA – either a linear or exponential equation that describes the growth over time.

GROWTH PATTERN – a set of values usually visualized by plotting points on a grid and fitting either a linear or an exponential equation to the scatter plot.

IDENTIFY - to choose from a set or to name cases in which the desired result is present or true.

INDEPENDENT VARIABLE - in the equation, $y = 3x + 2$, x is the independent variable and y is the dependent variable. Substituting any value for x leads to a value for y.

INDIRECT MEASUREMENT - a measurement that is impossible or impractical to be measured directly or physically, usually calculated using a formula or a known relationship.

INDIRECT TECHNIQUE - the method used to determine an indirect measurement.

INTEGER - any whole number or its opposite.

INTEGRAL COEFFICIENT - in the expression, $3x$, 3 is the coefficient. Integral coefficients are coefficients that are integers.

INTERPOLATE - determine a value within a set of given values using a formula, rule, or reasonable estimation.

INVERSE OPERATION - examples of inverse operations are addition and subtraction, multiplication and division, extracting a root and raising to a power.

INVERSE VARIATION - a variation of the type $xy = k$; the graph is a hyperbola with axes as asymptotes.

IRREGULAR SHAPES - shapes that are not one of the named geometric shapes or combinations of shapes; shapes that may contain curved portions rather than straight-line segments.

JOINT VARIATION - variation in which the values of one variable depend on two or more direct variations, $z = kxy$.

JUSTIFY - give a logical explanation or informal proof of a mathematical situation, computation or property.

LAW OF COSINES - for any triangle with sides a, b, c, and angles A, B, and C, a squared = b squared + c squared - 2bc Cos A.

LAW OF SINES - for any triangle with sides a, b, c and angles A, B, and C, sin A/a = sin B/b = sin C/c.

LINE SYMMETRY - a figure has line symmetry if there is a least one line that divides the figure into two parts that are mirror images of each other.

LINEAR EQUATION - an equation which has a straight line graph.

LOCAL BEHAVIOR - a description of the values of a function or relation within a small interval of the independent variable.

LOGARITHM - a logarithm of a number is the exponent to which a given base must be raised to produce the given number.

LOGARITHMIC GROWTH - a set of values which are approximated by an equation of the form y = log b x.

LOGARITHMIC NOTATION - use of the symbols "log" or "ln" in context.

LOWER BOUND - any number below which a function value may approach but not pass.

MANIPULATIVES - objects that can be arranged, built, and moved around by hand, (see concrete materials.)

MATHEMATICAL NOTATION - correct use of labels, symbols, and abbreviations in a mathematics context.

MIRA - a plastic device which is used to determine and complete symmetries by reflecting images and allowing the user to also see through the reflecting surface.

MIXED VARIATION - variation that contains both direct and inverse variation.

MODE - the most frequently occurring value or values in a set of values.

MODEL - to create, using concrete materials, drawings or symbols; a representation of a mathematical relationship or situation.

NATURAL NUMBER - the counting numbers, starting with one.

NAVIGATIONAL SYSTEM - compass directions or bearings in a variety of formats.

NET - a two-dimensional pattern that will fold into a three-dimensional figure.

NON-CONSTANT RATE OF CHANGE - set of data or table of values in which the amount of the dependent variable does not change by a constant value as the value of the independent variable changes by a constant value.

NON-DEGENERATE CONIC SECTIONS - see conic sections.

NON-REGULAR SHAPE - a shape that does not have all sides congruent and all angles congruent.

ORDER - to place numbers in order from smallest to largest or largest to smallest.

ORDER OF OPERATIONS - the rule for using operations on numbers; first parentheses, then exponents, then multiplication and division, then addition and subtraction.

PARAMETRIC EQUATIONS - two equations which express the coordinates of x and y as separate functions of a common variable, called the parameter that is usually time.

PERPENDICULAR BISECTOR - a line segment or ray that intersects a segment at its midpoint at a 90^{o} angle.

PIECEWISE FUNCTION - a function consisting of two or more equations, defined for specified intervals of the independent variable.

POLYHEDRON - a solid figure bounded by faces that are polygons; two faces meet at an edge, two or more edges meet at a vertex.

PRIME FACTORIZATION - the unique set of factors of a number, all of which are prime numbers.

PROBABILITY - a ratio of the number of desired outcomes and the number of possible outcomes.

PROBABILITY EVENTS - the set of all possible outcomes of an experiment is the sample space; any subset of the sample space is an event.

PROPERTIES OF EQUALITY - reflexive property (a = a), symmetric property (if a = b, then b = a), and transitive property (if a = b and b = c, then a = c).

PROPORTIONALITY - a relationship described by a constant ratio.

PROVE - use logical arguments, definitions, theorems, and properties to show that a relationship is true for all numbers or specific set of figures.

PYTHAGOREAN THEOREM - In a right triangle with sides a and b, and hypotenuse c, the theorem states that $c^2 = a^2 + b^2$.

PYTHAGOREAN TRIPLES - sets of three numbers that satisfy the Pythagorean theorem.

QUADRATIC EQUATION - any equation of the form $y = ax^2 + bx + c$.

RADIAN - an angle measure based on the premise that there are 2π radians in a complete circle.

RATIO - a comparison of one quantity to another.

RATIO OF SIMILITUDE - the simplest form ratio of the measures of corresponding parts of similar figures.

REAL NUMBERS - rational and irrational numbers.

RECOMPOSE - put addends or factors of a given number back together in a way different from the original arrangement or decomposition.

RECURSIVE FORM - a formula for the next term of a sequence given the term before it.

REFLECTION - a rigid motion of an object on the plane, also known as a "flip", where the object is replicated as if it were folded across a line and imprinted in its new location.

REGULAR POLYGON - a polygon with equal sides and equal angles.

REGULAR SHAPE - see regular polygon.

RIGID MOTION – a motion that preserves shape and size.

ROTATIONAL SYMMETRY - a figure has rotational symmetry if it can be rotated (turned) less than 360 degrees about a point so that it appears the same as the original figure.

ROTATION - a rigid motion in the plane in which a figure is rotated (turned) about a point in the plane.

SCIENTIFIC NOTATION - a notation for expressing very large and very small numbers as a product of a decimal number greater than or equal to one and less than ten and a power of ten.

SET OF DATA POINTS - data collected and placed into ordered pairs for the purpose of graphing.

SIMPLE POLYGONS - convex, closed shapes bounded by line segments joined end to end.

SIMPLE EVENTS - a single activity in a probability experiment such as flipping a coin.

SIMULATION - an experiment that has the same number of outcomes as a given situation but is easier or more practical to carry out than the given situation.

SLOPE - the ratio of the increase in the y-values to the increase in the x-values between any two ordered pairs.

SLOPE-INTERCEPT FORM - the form of a linear equation $y = mx + b$ where m represents the slope and b represents the y-intercept.

STANDARD FORM - the form of a number expressed as a sum of products involving powers of ten.

STEM AND LEAF PLOTS - a method of displaying data where the leading digit(s) are the stem and the ending single digits are arranged in ascending order to the side representing the leaves.

SUCCESSIVE APPROXIMATION - a sequence of approximations, each one closer to the desired value.

SURFACE AREA - the total area required to cover the surface of a three-dimensional object.

SYSTEM OF EQUATIONS - a set of two or more equations.

TABLE OF VALUES - a table of two columns, the first representing values of the independent variable, the second representing the values of the dependent variable.

TESSELLATION - a tiling of a plane with no gaps and no overlaps.

TRANSFORMATIONS - motions in the plane; translations, rotations, reflections, dilations and contractions.

TRANSLATION - a rigid motion of an object in the plane also known as a "slide" where the object moves from one location to another.

TREE DIAGRAM - a counting technique where outcome options are shown as "branches" and the total number of branches at the end of the tree represent the total number of possible outcomes.

TRIANGLE INEQUALITY - the property that states that the length of any side of a triangle is less than the sum of the lengths of the other two sides.

UNIT ANALYSIS - the process of using conversion factors to change from one measure or rate to another.

UNIT OF ATTRIBUTE - the unit chosen depends upon the attribute being measured (e.g., for the attribute "length", a unit might be "meters").

UNIT FRACTION – fraction with one as a numerator and a natural number as a denominator.

UPPER BOUND - any number above which a function value may approach but not pass.

VECTOR - a line segment with both direction and length.

VERTICAL ANGLES - non-adjacent angles formed by the intersection of two lines.

VERTICES - plural form of vertex; the point of intersection of the rays of an angle, "corner" point of any geometric figure bounded by lines, planes, or lines and planes.

VOLUME - the amount of space occupied in three dimensions.

RELATIONSHIP OF PERFORMANCE DESCRIPTORS TO NATIONAL AND STATE STANDARDS

The writing team began its work by examining the National Council of Teachers of Mathematics new publication, *Principles and Standards for School Mathematics*. Since this document had just been released after an intensive national review period, the writers agreed that it was a credible source on which to base the Illinois Performance Descriptors. Many of NCTM's standards were adapted to meet the needs of the Illinois Performance Descriptors and are designated by **. Also, statements that were quoted directly are identified with an *.

The writers also had other states' documents to consult as their work progressed. The documents used were from Colorado, Kansas, Nevada, North Carolina, Texas, and Virginia. The ACT *Standards for Transition* were also examined and incorporated where applicable.

Because of the variety of works reviewed, the writers feel that they have produced a comprehensive description of mathematics performance for Illinois' students.

REFERENCES

ACT, Inc. Standards for Transition: Descriptions of the Skills and Knowledge Associated with PLAN and ACT Assessment Scores. http://www.act.org/standard/planact/scores.html, 1997.

Bell, Max, and Others. Everyday Mathematics Student Reference Book. Chicago, IL: Everyday Learning Corporation, 1998.

Colorado Department of Education. Colorado Model Content Standards for Mathematics Suggested Grade Level Expectations. http://www.cde.state.co.us/index_stnd.htm, 2000.

Kansas Department of Education. Kansas Curricular Standards for Mathematics, http://www.ksbe.state.ks.us/outcomes/Mathpg.html, 1999.

Karush, William. Webster's New World Dictionary of Mathematics. New York, NY: Webster's New World, 1989.

National Council of Teachers of Mathematics. Principles and Standards for School Mathematics. Reston, VA: The National Council of Teachers of Mathematics, Inc., 2000.

Nevada Department of Education. Academic Standards for Mathematics. http://www.nsn.k12.nv.us/nvdoe/, 1999.

North Carolina Department of Public Instruction. Mathematics Curriculum. http://www.ncpublicschools.org/curriculum/mathematics/, 1999.

Texas Education Agency. Texas Essential Knowledge and Skills for Mathematics, http://www.tea.state.tx.us/rules/tac/ch111toc.html, 1997.

Virginia Board of Education. Mathematics Standards of Learning, http://www.pen.k12.va.us/go/Sols/math.html, 1995.