MATHEMATICS
PERFORMANCE DESCRIPTORS

GRADES 6-12

RESPONDING TO THIS DOCUMENT

We welcome your response to this document.

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INTRODUCTION

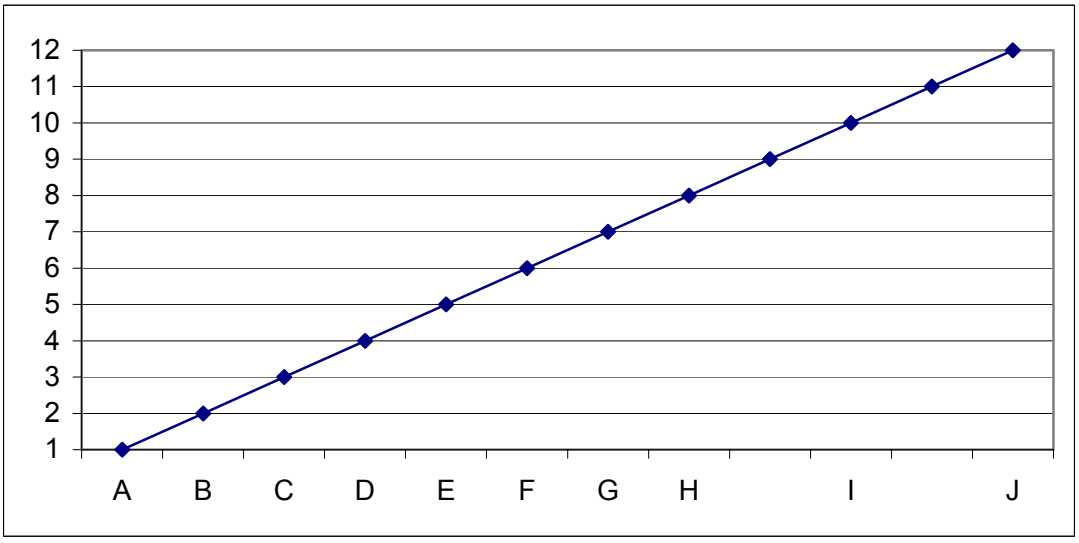
Design for Performance Standards

The Illinois Learning Standards are content standards that describe *"what"* students should know and be able to do in grades K – 12. Each content standard includes five benchmarks that describe what students should know and be able to do at early elementary, late elementary, middle/junior high, early high school, and late high school.

The challenge for the 2000-2001 school year was to produce performance standards that would indicate *"how well"* students should perform to meet the standards. To address this challenge, a number of perspectives needed to be considered. For example, the National Governors Association[1] raised two pertinent questions policymakers should consider for the design of performance standards:

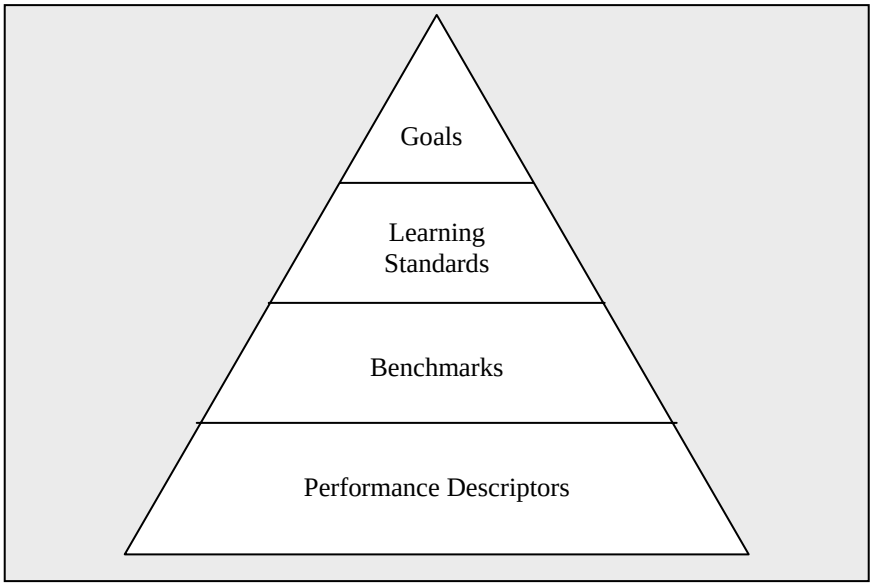
- Do the performance standards indicate the levels of performance students should attain, descriptions of performance at each level, and rules that enable educators to determine whether students have reached a given level?
- Do the performance standards include a range of work . . . to show that students can meet the standards in a variety of ways?

The performance standards describe how well students perform at various points on an educational development continuum. This continuum shows how students can demonstrate mastery of progressively more difficult content and cognitive skills over ten incremental stages of development. Performance within each stage can be assessed by the extent to which students are meeting the standards (i.e., starting, approaching, meeting, exceeding). Performance standards include four essential elements: performance descriptors, performance levels, assessment tasks, and performance examples.



[1] Ouellette, M. (2000). "Maintaining progress through systemic education reform: Performance standards," Washington, DC: National Governors Association.

The performance standards are classroom resources for voluntary use at the local level. They are not intended to replace the Illinois Learning Standards. Instead, they supplement them by providing sufficient detail and examples to enable teachers to establish appropriate grade-level performance expectations for students. The performance descriptors are a direct outgrowth of the state goals for learning. Whereas the benchmarks filled in detail on each of the standards at five grade-level clusters, the performance descriptors provide additional detail at each grade level.



Definitions

performance standards*:* the knowledge and skills that students are to perform at various stages of educational development (*performance descriptors*) and the performance expectations (*performance levels and assessment tasks)* for student work *(performance exemplars)* at each of the stages.

performance descriptors*:* statements of how students can demonstrate the knowledge and skills they acquired.[2]

performance levels*:* descriptions of how well students have achieved the standards; that is, the range, frequency, facility, depth, creativity, and/or quality of the knowledge and skills they acquired. Students can demonstrate levels of achieving performance standards along six dimensions:

PERFORMANCE LEVEL =	**RANGE +**	**FREQUENCY +**	**FACILITY +**	**DEPTH +**	**CREATIVITY +**	**QUALITY**
Exceeding	extensively	consistently	automatically	profoundly	inventively	excellently
Meeting	fully	usually	quickly	deeply	imaginatively	well
Approaching	partially	occasionally	haltingly	cursorily	commonly	marginally
Starting	narrowly	rarely	slowly	superficially	imitatively	poorly

[2] New Standards. Performance Standards. (1997) Washington, DC: The National Center on Education and the Economy.

assessment tasks: descriptions of what students can do to demonstrate they have met the standards and a means for evaluating the levels of their performance.

performance examples: student work samples resulting from the classroom-based assessment tasks that illustrate performance levels.

Template For Expanded Performance Descriptors

BENCHMARKS ⇒	early elementary			late elementary		middle/ junior high			early high	late high
STAGES ⇒ PERFORMANCE LEVELS ⇓	A	B	C	D	E	F	G	H	I	J
Exceeding										
Meeting										
Approaching										
Starting										

Vision for Mathematics Performance

Students need to see mathematics as a language, a tool, and an art form with which they can communicate ideas, solve problems, and explore the world around them. By the end of twelfth grade, they will have been encouraged to see multiple ways of expressing mathematical ideas, guided to make multiple connections to real life situations, and prompted to work with others as they explore possibilities. Their thinking will have become flexible, transferring mathematical knowledge from one situation to another, and they will be able to communicate their thinking in a variety of ways using any technology available to them.

Students will make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures. Through this process, they will develop mathematical reasoning skills and become adept at evaluating their own thinking. Students will have an understanding of how numbers are used and represented. They will be able to estimate and use basic operations to both solve everyday problems and confront more involved calculations in algebraic and statistical settings. They will be able to read, write, visualize, and talk about ways in which mathematical problems can be solved in both theoretical and practical situations. They will be able to communicate relationships in geometric and statistical settings through drawings and graphs.

Goal 6 – Demonstrate and apply a knowledge and sense of numbers, including numeration and operations (addition, subtraction, multiplication, division), patterns, ratios, and proportions. Students must have an understanding of and proficiency with counting, numbers and arithmetic, number systems and their structures. Development of number sense is central to the students' mathematical understanding. They must have the ability to compose, decompose, and recompose numbers, use the relationships among arithmetic operations to solve problems, and understand the base-ten number system. By the end of twelfth grade, students should be able to estimate, make sense of numbers, and recognize that the ability to determine relative and absolute magnitudes of numbers are important mathematical skills. The students must attain a rich understanding of the role of numbers: what they are; how they are

represented with objects, numerals, or on number lines; how they are related to one another; how numbers are embedded in systems that have structures and properties; and how to use numbers and operations to solve problems. Computational fluency of basic number combinations is essential, especially knowing the best tools to use for estimating and computing in a variety of situations: mental math, paper and pencil, and technology. The students must be able to explain their methods, understand that many methods exist, and see the usefulness of methods that are efficient, accurate and applicable.

Goal 7 – Estimate, make, and use measurements of objects, quantities, and relationships and determine acceptable levels of accuracy. Students must develop an understanding of what is a measurable attribute and become familiar with the units and processes that are used in measuring. The tools, techniques, and formulas used in measuring must be developed in a variety of situations. The sets of attributes that students measure should expand as they progress: length, perimeter, area, volume, temperature, time, and angle measure. They should explore derived measurement such as speed, understand the role of units, and know how scale affects measurement. Students must see that all measurements are approximate, and that estimation skills play a significant role in life.

Goal 8 – Use algebraic and analytical methods to identify and describe patterns and relationships in data, solve problems, and predict results. Students must begin at an early age exploring patterns to build toward the understanding of functions. Their experience with numbers and patterns will form a foundation for symbols, properties, and algebraic expressions. Students must be able to use algebraic methods to construct and examine tables of values; to interpret the relationships expressed by patterns in these tables; to represent patterns in multiple ways including graphs and formulas; to reason about changes in quantities and the relationships involved in changes; and to find solutions to everyday problems using the tools of algebra and logic.

Goal 9 – Use geometric methods to analyze, categorize, and draw conclusions about points, lines, planes, and space. Students begin by learning about geometric shapes and how to analyze the characteristics and relationships by building and manipulating representations of two- and three-dimensional objects, as well as perceiving objects from different perspectives. Geometric modeling and spatial reasoning offer ways for students to interpret and describe physical environments (e.g., maps, floor plans, art) and can be important tools in problem solving. Students must make connections between number patterns, geometry, and algebra (e.g., area representations of fractions, algebraic and geometric representations of number patterns). Students must make and explore conjectures about geometry and learn to reason carefully about geometric ideas. They must build an understanding across the grades, from informal to more formal thinking, developing the ideas of reasoning and proof. Technology plays an important role by facilitating many ways to explore conjectures.

Goal 10 – Collect, organize, and analyze data using statistical methods; predict results; and interpret uncertainty using concepts of probability. Students must be able to organize data; identify patterns in the data; and judge, using logic, the reasonableness of any claims and interpretations. Students must collect, display, and interpret data from a variety of sources (e.g., opinion polls, stock prices, tax rates, crime statistics, scientific studies, weather reports) to answer specific questions. They also must construct, analyze, and evaluate arguments that involve data, interpretation, and logical predictions. All students need to understand and apply the role probability plays in decision-making.

The ISBE Mathematics Expanded Performance Descriptors:

Intended Use and Interpretation

When the task force charged with writing the expanded performance descriptors for mathematics met and discussed the task, we were excited about the possibilities and the potential for helping teachers. In order for the materials to meet their potential, there are several key points the writers would like to bring to your attention before you start looking at this document and begin to implement changes in the classroom that reflect these expectations for students. What follows is important to the intended use and implementation of this document.

1) **Different goals receive different amounts of emphasis each year.** The Third International Mathematics and Science Study (TIMSS) data has shown us that the U.S. has traditionally tried to teach every area of mathematics every year, resulting in a curriculum that is "a mile wide and an inch deep". As a result, it has often been recommended that we teach fewer topics each year and teach them in greater depth. The NCTM *Principles and Standards for School Mathematics* (2000) are still grouped by grade bands, not individual grade levels, in part, because of this very concern. It also has a section in the beginning of chapter three dedicated to this topic, entitled “Growth across the Grades: Aiming for Focus and Coherence” (p. 30). The following is a short excerpt from that document.

> The Table of Standards and expectations in the appendix highlights the growth of expectations across the grades. It is not expected that every topic will be addressed each year. Rather, students will reach a certain depth of understanding of the concepts and acquire certain levels of fluency with the procedures by prescribed points in the curriculum, so further instruction can assume and build on this understanding and fluency.
>
> Even though each of these ten Standards applies to all grades, emphases will vary both within and between the grade bands. For instance, the emphasis on number is greatest in pre-kindergarten through grade two, and by grades nine through twelve, number receives less instructional attention. And the total time for mathematical instruction will be divided differently according to particular needs in each grade band—for example, in the middle grades, the majority of instructional time would address algebra and geometry. Figure 3.1 shows roughly how the Content Standards might receive different emphases across the grade bands.

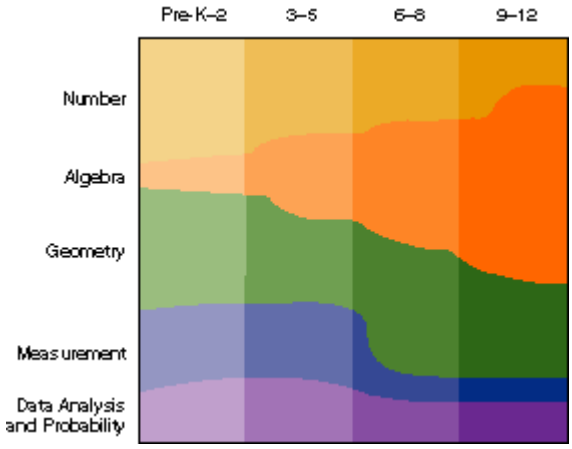


Fig. **3.1.** The Content Standards should receive different emphases across the grade bands (NCTM, 2000, p.30).

2) **The document is intended to be a developmental guide.** Stages are not intended to define grade levels. Several stages correspond to levels where ISAT will occur, and the performance descriptors match the "meets" standards at those levels. However, one stage below or above may not represent one grade level below or above. Instead, they represent the developmental stages of student learning, and show a progression through which students develop understanding of mathematical concepts. As a result, it is not enough for a teacher to look at a single stage and decide what content he or she will teach that year. He or she must look at a series of three stages to see the progression of understanding students should experience in order to move from one ISAT stage to the next. Stage C **does** correspond to the third grade ISAT expectations, Stage E to fifth grade, Stage H to eighth grade, Stage I to early high school, and Stage J to late high school. The other stages **are not** meant to explicitly correspond to the missing grades between.

3) **The document is a curriculum development tool.** The stages of development presented in this document can help a school district develop curriculum that will meet state standards. By using the stages corresponding to ISAT, curricula can move students through the stages of development without teaching every topic every year. For instance, when circle graphs are listed as a descriptor in Goal 10, Stage F, it means that the topic should be mastered before going on to further study of data displays and interpretation that will occur in Stage G and Stage H. There are multiple ways this could be handled. For example:
   - It is very possible that all three stages F, G, and H could be accomplished within a single year of study at seventh grade, if the seventh grade year were devoted to the study of three of four major topics, one being statistics.
   - Another possibility is combining circle graphs with the study of percent in grade six, where a focus of the entire school year could be rational numbers, ratios, percents, and proportions. All mathematical content taught would then relate to these ideas, including the study and use of graphs, geometry (such as similar figures), or probability to help present fractions and ratios. So while many areas of mathematics are included, there are not separate units on geometry or statistics, but some of these ideas are incorporated into the main themes of the school year.

In this manner, the EPDs are used to help develop a curriculum rather than to prescribe a state-mandated curriculum of what should be taught at each grade.

4) **It is intended that each stage represent growth from the previous stage.** An idea is repeated in a stage only if new content or concepts are introduced with respect to that topic. For instance, once the concept of multi-digit addition and subtraction is placed in stage B, it is not repeated in stage C or above. It is assumed that students will expand on their knowledge and skills in the use of addition and subtraction, but it is not a new concept to be mastered. This is designed to remove isolated review from the curriculum. All review should be done in context of the new content being presented. For example, it is intended that there be no direct teaching of whole number computation beyond Stage D. By the time students reach Stage E, they should have mastered whole number computation. Similarly, all computation with rational numbers should be introduced and taught during stage E through stage H, and no computation with rational numbers should be directly taught after 8th grade. The writers realize that students who are below standards will need remedial work and extra help beyond the regular instructional time.

5) **The document uses appropriate mathematical terminology.** Occasionally, a reader may encounter an unfamiliar term. For example, in the early stages of geometry instruction the terms “slides”, “flips”, and “turns” are often used to describe translations, reflections, and rotations. It is our hope that a vocabulary shift will occur and the correct mathematical terms will be used. In order to assist the reader with terminology used in this document, a glossary has been written with specific mathematical terms, as well as verbs like "develop" to clarify what is meant by developing a concept or an idea.

6) **The document is based on the current form of the state learning goals and their benchmarks.** It is assumed that the state goals and benchmarks will undergo continual refinement, and this document will need to be revised as that happens. This document has also been carefully aligned with the NCTM *Principles and Standards for School Mathematics* as indicated by * and **. All descriptors are written to indicate the "meets" level of performance for each stage and are not intended to limit instruction. Curriculum should be fluid and change as needed. As such, this document should always be considered a work in progress.

Mathematics Performance Descriptors

6A Students who meet the standard can demonstrate knowledge and use of numbers and their many representations in a broad range of theoretical and practical settings. *(Representations)*

Stage E	Stage F	Stage G
1. Place mixed numbers and decimals on a number line. 2. Show equivalent representations of a number by changing from one form to another form (e.g., standard form to expanded form, fraction to decimal, decimal to percent, improper fraction to mixed number). 3. Differentiate how fractions are used (part of a whole, part of a set, location on a number line, and division of a whole number). 4. Analyze how the size of the whole affects the size of the fraction (e.g., 1/2 of a large pizza is not the same as 1/2 of a small pizza). 5. Describe integers using familiar applications (e.g., a thermometer, above/below sea level).	1. Represent place values from units through billions using powers of ten. 2. Represent, order, compare, and graph integers. 3. Identify fractional pieces that have the same value but different shapes. 4. Compare and order fractions and decimals efficiently and find their approximate position on a number line. ** 5. Represent repeated factors using exponents.	1. Represent any large number using scientific notation. 2. Show relationships between sets of numbers, including rational numbers, whole numbers, natural numbers, and integers.
Grade 6 (E-F-G) Grade 7 (F-G-H) Grade 8 (G-H-I) Grade 9-10 (H-I-J) Grade 11-12 (I-J)		

* National Council of Teachers of Mathematics. *Principles and Standards for School Mathematics*. Reston, Va: National Council of Teachers of Mathematics, 2000.
** Adapted from: National Council of Teachers of Mathematics. *Principles and Standards for School Mathematics*. Reston, Va: National Council of Teachers of Mathematics, 2000.

Mathematics Performance Descriptors

6A Students who meet the standard can demonstrate knowledge and use of numbers and their many representations in a broad range of theoretical and practical settings. *(Representations)*

Stage H	Stage I	Stage J
1. Recognize and use exponential, scientific, and calculator notation. ** 2. Represent, order, and compare rational numbers using a variety of methods and materials. 3. Place rational numbers on a number line.	1. Illustrate the relationship between second and third roots and powers of a number. 2. Organize problem situations using matrices. 3. Represent, order, and compare real numbers. 4. Place real numbers on a number line.	1. Represent numbers in equivalent forms (e.g., exponential/logarithmic, radical/rational exponents). 2. Graph or interpret the graph of a complex number in rectangular and vector forms. 3. Represent numerical intervals using correct notation.
Grade 6 (E-F-G) Grade 7 (F-G-H) Grade 8 (G-H-I) Grade 9-10 (H-I-J) Grade 11-12 (I-J)		

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Mathematics Performance Descriptors

6B Students who meet the standard can investigate, represent, and solve problems using number facts, operations and their properties, algorithms, and relationships. (*Operations and properties)*

Stage E	Stage F	Stage G
1. Determine whether a number is prime or composite. 2. Identify all the whole number factors of a composite number. 3. Explore and identify properties of square numbers. 4. Compute with 10, 100, 1000, and other powers of 10. 5. Explore and use divisibility rules. 6. Solve number sentences and word problems using addition and subtraction of fractions with unlike denominators. 7. Solve number sentences and word problems using addition and subtraction of decimals.	1. Write prime factorizations of numbers. 2. Determine the least common multiple and the greatest common factor of a set of numbers. 3. Demonstrate the meaning of multiplication of fractions (e.g.,1/2 x 3 is 1/2 of a group of three objects). 4. Simplify simple arithmetic expressions with rational numbers using the field properties and the order of operations. 5. Recognize and use the inverse relationships of addition and subtraction, multiplication and division to simplify computations and solve problems. ** 6. Solve multiplication number sentences and word problems with whole numbers and familiar fractions.	1. Write prime factorizations using exponents. 2. Describe relationships between prime factorizations and properties of squares, primes, and composites. 3. Classify numbers according to the number of whole number factors (e.g., square numbers have an odd number of factors). 4. Demonstrate and describe the effects of multiplying or dividing by a fraction less than or greater than one. 5. Simplify arithmetic expressions containing exponents using the field properties and the order of operations. 6. Justify rules of divisibility for 2, 5, and 10. 7. Solve multi-step number sentences and word problems with rational numbers using the four basic operations.
Grade 6 (E-F-G) Grade 7 (F-G-H) Grade 8 (G-H-I) Grade 9-10 (H-I-J) Grade 11-12 (I-J)		

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Mathematics Performance Descriptors

6B Students who meet the standard can investigate, represent, and solve problems using number facts, operations and their properties, algorithms, and relationships. (*Operations and properties)*

Stage H	Stage I	Stage J
1. Determine the least common multiple and greatest common factor of a set of numbers using prime factorization containing exponents. 2. Determine and describe the effects of arithmetic operations with decimals and integers (e.g., multiply by a decimal between zero and one, divide by a negative integer). 3. Simplify arithmetic expressions containing integers using the field properties and order of operations. 4. Describe and use the inverse relationships of squaring and finding square roots to simplify computations and solve problems. ** 5. Justify divisibility rules for 3, 4, 6, 8, and 9.	1. Compare and contrast the properties of numbers and number systems, including the rational and the real numbers. ** 2. Determine an appropriate numerical representation of a problem situation, including roots and powers, if applicable. 3. Judge the effects of such operations as multiplication, division, and computing powers and roots on the magnitudes of quantities. * 4. Solve problems using simple matrix operations (addition, subtraction, scalar multiplication). 5. Develop fluency in operations with real numbers using mental computation or paper-and-pencil calculations for simple cases and technology for more-complicated cases. ** 6. Judge the reasonableness of numerical computations and their results. *	**1.** Compare and contrast the properties of numbers and number systems, including the complex numbers as solutions to quadratic equations that do not have real solutions. ** 2. Simplify expressions using the field properties, order properties, and properties of equality for the set of real numbers. 3. Use the field properties and properties of equality for the set of complex numbers. 4. Determine the opposite, reciprocal, absolute values, and positive integral powers of a complex number. 5. Identify, represent, and solve problems with numbers expressed in exponential, logarithmic, and scientific notations using technology. 6. Solve problems using exponents and logarithms. 7. Solve problems using complex numbers and their various representations. 8. Explain that vectors and matrices are systems that have some of the properties of the real-number system. ** 9. Solve problems using matrices. **10.** Develop fluency in operations with real numbers, vectors, and matrices using mental computation or paper-and-pencil calculations for simple cases and technology for more-complicated cases. *
Grade 6 (E-F-G) Grade 7 (F-G-H) Grade 8 (G-H-I) Grade 9-10 (H-I-J) Grade 11-12 (I-J)		

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Mathematics Performance Descriptors

6C Students who meet the standard can compute and estimate using mental mathematics, paper-and-pencil methods, calculators, and computers. *(Choice of method)*

Stage E	Stage F	Stage G
1. Develop and use strategies to estimate computations involving familiar fractions and decimals in situations relevant to students' experience * (e.g., double a recipe with 3/8 cup sugar, will more than a cup of sugar be needed). 2. Evaluate estimates to judge their reasonableness and degree of accuracy. 3. Select and use appropriate operation(s) and tool(s) (e.g., mental math, pencil-and-paper, estimation, calculator, computer) to perform calculations on whole numbers, fractions and decimals according to the context and nature of the computation. ** 4. Determine and justify whether exact answers or estimates are appropriate.	1. Select and use appropriate operations, methods, and tools to compute or estimate using whole numbers with natural number exponents. ** 2. Analyze algorithms for computing with whole numbers, familiar fractions, and decimals and develop fluency in their use. **	1. Select, use, and justify appropriate operations, methods, and tools to compute or estimate with integers and familiar rational numbers. ** 2. Develop, use, and explain strategies to compute exact answers mentally with integers and simple rational numbers using a variety of techniques (e.g., estimate and compensate, halve and double, compatible numbers, decomposition and recomposition using the distributive property). 3. Analyze algorithms for computing with rational numbers and develop fluency in their use. **
Grade 6 (E-F-G)	Grade 7 (F-G-H)	Grade 8 (G-H-I)	Grade 9-10 (H-I-J)	Grade 11-12 (I-J)

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Mathematics Performance Descriptors

6C Students who meet the standard can compute and estimate using mental mathematics, paper-and-pencil methods, calculators, and computers. *(Choice of method)*

Stage H	Stage I	Stage J
1. Select, use, and justify appropriate operations, methods, and tools to compute or estimate with real numbers. ** 2. Analyze algorithms for computing with real numbers and develop fluency in their use. **	1. Develop fluency in operations with real numbers and matrices using mental computation or paper-and-pencil calculations for simple cases and technology for more-complicated cases. ** 2. Determine and explain whether exact values or approximations are needed in a variety of situations. 3. Determine an appropriate number of digits to represent an outcome.	1. Determine the level of accuracy needed for computations involving measurement and irrational numbers. 2. Use the correct number of digits in computation to achieve an appropriate unit or level of accuracy when solving problems. 3. Estimate an appropriate answer for a given term of a sequence. 4. Describe the role of rounding error in calculations.
Grade 6 (E-F-G) Grade 7 (F-G-H) Grade 8 (G-H-I) Grade 9-10 (H-I-J) Grade 11-12 (I-J)		

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Mathematics Performance Descriptors

6D Students who meet the standard can solve problems using comparison of quantities, ratios, proportions, and percents.

Stage E	Stage F	Stage G
1. Identify and express ratios using appropriate notation (i.e., a/b, a to b, a:b). 2. Model the concept of percent using manipulatives or drawings.	1. Solve number sentences and word problems using percents. 2. Demonstrate and explain the meaning of percents, including greater than 100 and less than 1. ** 3. Create and explain a pattern that shows a constant ratio. 4. Analyze situations to determine whether ratios are appropriate to solve problems. 5. Determine equivalent ratios.	1. Work flexibly with fractions, decimals, and percents to solve number sentences and word problems (e.g., 50% of 10 is the same as 1/2 of 10 is the same as 0.5 x 10). ** 2. Create and explain ratios and proportions that represent quantitative relationships. 3. Create and explain a variety of equivalent ratios to represent a given situation. 4. Develop, use, analyze, and explain methods for solving numeric or word problems involving proportions. **
Grade 6 (E-F-G) Grade 7 (F-G-H) Grade 8 (G-H-I) Grade 9-10 (H-I-J) Grade 11-12 (I-J)		

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Mathematics Performance Descriptors

6D Students who meet the standard can solve problems using comparison of quantities, ratios, proportions, and percents.

Stage H	Stage I	Stage J
1. Develop, use, analyze, and explain methods for solving number sentences or word problems involving proportions with rational numbers. ** 2. Solve problems that involve percents, including percent increase and decrease, regardless of the piece of information that is missing.	1. Explain how ratios and proportions can be used to solve problems of percent, growth, and error tolerance. 2. Set up and solve proportions for direct and inverse variation of simple quantities.	1. Explain the connection of percents to growth patterns, error, and probability. 2. Set up and solve proportions for direct, inverse, and compound variations of quantities involving powers and multiple variables.
Grade 6 (E-F-G) Grade 7 (F-G-H) Grade 8 (G-H-I) Grade 9-10 (H-I-J) Grade 11-12 (I-J)		

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Mathematics Performance Descriptors

7A Students who meet the standard can measure and compare quantities using appropriate units, instruments, and methods. *(Performance and conversion of measurements)*

Stage E	Stage F	Stage G
1. Convert U.S. customary and metric measurements into larger or smaller units. 2. Draw an angle of any given measure using a protractor or angle ruler.	1. Investigate the history of the U.S. customary and metric systems of measurement. 2. Measure, with a greater degree of accuracy, any angle using a protractor or angle ruler.	1. Select and justify the choice of either U.S. customary or metric systems of measurement according to the situation (e.g., measure fabric in yards, measure dry chemicals in grams). 2. Make simple measurements to determine indirect measures (e.g., determining the height of a flagpole using its shadow and similar right triangles).
Grade 6 (E-F-G) Grade 7 (F-G-H) Grade 8 (G-H-I) Grade 9-10 (H-I-J) Grade 11-12 (I-J)		

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Mathematics Performance Descriptors

7A Students who meet the standard can measure and compare quantities using appropriate units, instruments, and methods. *(Performance and conversion of measurements)*

Stage H	Stage I	Stage J
1. Solve simple scale conversions, contractions, and dilations (e.g., maps and diagrams).	1. Select units and scales that are appropriate for problem situations involving measurement. ** 2. Convert between the U.S. customary and metric systems given the conversion factor.	1. Convert angle measures between degrees and radians. 2. Set up and solve measurement conversions using multiple rates and conversion factors.
Grade 6 (E-F-G) Grade 7 (F-G-H) Grade 8 (G-H-I) Grade 9-10 (H-I-J) Grade 11-12 (I-J)		

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Mathematics Performance Descriptors

7B Students who meet the standard can estimate measurements and determine acceptable levels of accuracy. *(Estimation)*

Stage E	Stage F	Stage G
1. Explain that all measurements are approximations. 2. Describe how precision is affected by choice of units. 3. Estimate the perimeter, area, and/or volume of regular and irregular shapes and objects.	1. Estimate distance, weight, temperature, and elapsed time using reasonable units and with acceptable levels of accuracy.	1. Estimate angle measure, area, and volume using reasonable units and with acceptable levels of accuracy. 2. Determine and describe acceptable levels of accuracy in estimation situations.
Grade 6 (E-F-G) Grade 7 (F-G-H) Grade 8 (G-H-I) Grade 9-10 (H-I-J) Grade 11-12 (I-J)		

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Mathematics Performance Descriptors

7B Students who meet the standard can estimate measurements and determine acceptable levels of accuracy. *(Estimation)*

Stage H	Stage I	Stage J
1. Measure any quantity to the greatest degree of accuracy determined by the tool. 2. Determine the maximum error in measurements.	1. Estimate the magnitude and directions of physical quantities (e.g., velocity, force, slope). 2. Determine answers to an appropriate degree of accuracy using significant digits.	1. Analyze precision, accuracy, and approximate error in measurement situations. 2. Determine a reasonable estimate of measure for more complex problem situations. 3. Solve problems to a desired interval of accuracy. 4. Apply informal concepts of successive approximation, upper and lower bounds, and limit in measurement situations.
Grade 6 (E-F-G) Grade 7 (F-G-H) Grade 8 (G-H-I) Grade 9-10 (H-I-J) Grade 11-12 (I-J)		

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Mathematics Performance Descriptors

7C Students who meet the standard can select and use appropriate technology, instruments, and formulas to solve problems, interpret results, and communicate findings. *(Progression from selection of appropriate tools and methods to application of measurements to solve problems)*

Stage E	Stage F	Stage G
1. Select appropriate tools to measure, draw, or construct figures. 2. Develop and discuss strategies for determining area and perimeter of irregular shapes. 3. Explain the meaning of a measurement answer in context. 4. Read and interpret a scale on a map or a scale drawing using the idea of a constant ratio (e.g., 1" represents 1 mile), and use it to answer questions about actual measurement. 5. Develop and use formulas to determine the area of squares, rectangles, and right triangles.	1. Select and justify an appropriate formula to find the area of triangles, parallelograms, and trapezoids. ** 2. Select an appropriate formula or strategy to find the surface area and volume of rectangular and triangular prisms. ** 3. Develop and use formulas for determining the area of triangles, parallelograms, and trapezoids. 4. Develop and use the formula for determining the volume of a rectangular and triangular prism. 5. Calculate the surface area of a cube, rectangular prism, and triangular prism. 6. Develop and use formulas for determining the circumference and area of circles.	1. Select and use appropriate units and tools to measure volume, surface area, and mass/weight accurately for a given situation. ** 2. Select an appropriate formula to determine the circumference and the area of circles. ** 3. Select and explain an appropriate formula or strategy to find the surface area and volume of rectangular and triangular pyramids, cylinders and cones. ** 4. Solve simple problems involving rate, time, and distance. 5. Solve problems involving mixed units of the same attribute, including time, money, length, and area. 6. Explore and explain derived measurements (e.g., velocity and density). 7. Develop and discuss strategies to find the area of combined shapes. **
Grade 6 (E-F-G) Grade 7 (F-G-H) Grade 8 (G-H-I) Grade 9-10 (H-I-J) Grade 11-12 (I-J)		

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Mathematics Performance Descriptors

7C Students who meet the standard can select and use appropriate technology, instruments, and formulas to solve problems, interpret results, and communicate findings. *(Progression from selection of appropriate tools and methods to application of measurements to solve problems)*

Stage H	Stage I	Stage J
1. Solve simple problems involving rates and other derived measurements such as velocity and density. **	1. Solve problems using indirect measurement by choosing appropriate technology, instruments, and/or formulas.	1. Solve practical problems using non-linear scales.
2. Solve problems involving angle measurement in polygons and circles.	2. Check measurement computations using unit analysis. **	
3. Develop and describe surface area and volume formulas for cones and cylinders by relating pyramids to cones and prisms to cylinders.	3. Describe the general trends of how the change in one measure affects other measures in the same figure (e.g., length, area, volume).	
4. Solve problems involving time, temperature, mass, speed, distance, density, and monetary values.	4. Determine linear measures, perimeters, areas, surface areas, and volumes of similar figures using the ratio of similitude.	
5. Solve problems involving scale drawings, models, maps, or blueprints.	5. Determine the ratio of similar figure perimeters, areas, and volumes using the ratio of similitude.	
6. Determine derived measurements.	6. Calculate by an appropriate method the length, width, height, perimeter, area, volume, surface area, angle measures, or sums of angle measures of common geometric figures, or combinations of common geometric figures.	
7. Determine the surface area of three-dimensional figures.	7. Solve problems involving multiple rates, measures, and conversions.	
8. Determine the volume of a sphere.		
Grade 6 (E-F-G) Grade 7 (F-G-H) Grade 8 (G-H-I) Grade 9-10 (H-I-J) Grade 11-12 (I-J)		

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Mathematics Performance Descriptors

8A Students who meet the standard can describe numerical relationships using variables and patterns. *(Representations and algebraic manipulations)*

Stage E	Stage F	Stage G
1. Describe, extend, and make generalizations about given geometric and numeric patterns. ** 2. Describe a pattern, with at least two operations, verbally and symbolically, given a table of input/output numbers. 3. Demonstrate equality of two expressions with variables (e.g., 28 + 35 = 35 + *n*). 4. Describe situations involving inverse relationships (e.g., the more people, the fewer cookies per person).	1. Investigate, extend, and describe arithmetic and geometric sequences of numbers whether presented in numeric or pictorial form. ** 2. Evaluate algebraic expressions for given values. 3. Express properties of numbers and operations using variables (e.g., the commutative property is m + n = n + m). 4. Simplify algebraic expressions involving like terms.	1. Investigate, describe, and generalize a variety of patterns using variable or recursive techniques. ** 2. Represent situations using variables. 3. Recognize and generate equivalent forms of simple algebraic expressions. **
Grade 6 (E-F-G) Grade 7 (F-G-H) Grade 8 (G-H-I) Grade 9-10 (H-I-J) Grade 11-12 (I-J)		

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Mathematics Performance Descriptors

8A Students who meet the standard can describe numerical relationships using variables and patterns. *(Representations and algebraic manipulations)*

Stage H	Stage I	Stage J
1. Investigate and describe linear, quadratic, and exponential patterns recursively. ** 2. Investigate and write algebraic expressions to describe the *nth* term of a simple linear, power, or exponential sequence. 3. Determine a specific term of a pattern of numbers or drawings. 4. Create arithmetic and geometric sequences to fit a given set of conditions. 5. Recognize and generate equivalent forms for linear equations, including transforming linear equations into standard and slope-intercept form. **	1. Write equivalent forms of equations, inequalities, and systems of equations. ** 2. Represent and explain mathematical relationships using symbolic algebra. ** 3. Model and describe slope as a constant rate of change. 4. Explain the difference between constant and non-constant rate of change. 5. Create an equation of a line of best fit from a set of ordered pairs or set of data points. 6. Simplify algebraic expressions using a variety of methods, including factoring. 7. Justify the results of symbol manipulations, including those carried out by technology. ** 8. Identify essential quantitative relationships in a situation and determine the class or classes of functions (e.g., linear, quadratic) that might model the relationships. ** 9. Represent relationships arising from various contexts using algebraic expression. 10. Rewrite absolute value inequalities in terms of two separate equivalent inequalities with the appropriate connecting phrase of "AND" or "OR".	1. Generalize patterns using explicitly-defined and recursively-defined sequences. ** 2. Translate between explicit and recursive forms of sequences where possible. 3. Represent relationships arising from various contexts using symbolic expressions, including iterative and recursive forms. 4. Symbolize growth patterns using variables. 5. Explain the differences and similarities of different forms of growth formulas. 6. Describe the limiting process using numerical analysis, graphs, and algebra. 7. Simplify algebraic expressions using exponential, logarithmic, and rational number techniques, including more advanced factoring.
Grade 6 (E-F-G) Grade 7 (F-G-H) Grade 8 (G-H-I) Grade 9-10 (H-I-J) Grade 11-12 (I-J)		

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Mathematics Performance Descriptors

8B Students who meet the standard can interpret and describe numerical relationships using tables, graphs, and symbols. *(Connections of representations including the rate of change)*

Stage E	Stage F	Stage G
1. Model problem situations with objects and equations to draw conclusions. ** 2. Represent and analyze patterns and functions using words, tables, and graphs. * 3. Demonstrate how the change in one quantity affects the other in a functional relationship involving whole numbers and unit fractions. 4. Identify, describe, and compare situations with constant and varying rates of change using words, tables, and graphs (e.g., two quantities that vary together are the length of the side of a square and its area). **	1. Graph simple inequalities on a number line. 2. Create a table of values that satisfy a simple linear equation and plot the points on the Cartesian plane. 3. Describe, verbally, symbolically, and graphically, a simple relationship presented by a set of ordered pairs of numbers.	1. Create a table of values that satisfy a power or exponential relationship and plot the points on the Cartesian plane. 2. Graph two inequalities with a single variable, including the intersection or union of these inequalities, on a number line.
Grade 6 (E-F-G) Grade 7 (F-G-H) Grade 8 (G-H-I) Grade 9-10 (H-I-J) Grade 11-12 (I-J)		

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Mathematics Performance Descriptors

8B Students who meet the standard can interpret and describe numerical relationships using tables, graphs, and symbols. *(Connections of representations including the rate of change)*

Stage H	Stage I	Stage J
1. Graph linear equations and inequalities on the Cartesian plane. 2. Graph a set of points and describe the relationship as linear or nonlinear. 3. Describe the relationships between symbolic expressions and graphs of lines using the appropriate vocabulary for the intercepts and slope of the line. ** 4. Graph absolute values on a number line. 5. Determine the slope of a line from a graph.	1. Describe the relationships of the independent and dependent variables from a graph. 2. Interpret the role of the coefficients and constants on the graph of linear and quadratic functions given a set of equations. 3. Relate the effect of translations on linear graphs and equations. 4. Create and connect representations that are tabular, graphical, numeric, and algebraic from a set of data. 5. Recognize and describe the general shape and properties of the graphs of linear, absolute value, and quadratic functions. 6. Approximate and interpret rates of change from graphical and numerical data. * 7. Identify slope in an equation and from a table of values. 8. Graph absolute values of linear functions on the Cartesian plane. 9. Recognize direct variation, inverse variation, linear, and exponential curves from their graphs, a table of values, or equations. ** 10. Interpret and use functions as a geometric representation of linear and non-linear relationships.	1. Fit an equation to data using a calculator. 2. Interpret the overall relationship of two variables and connect it to one of the function families (linear, exponential, logarithmic or power) from a graph. 3. Relate the effect of transformations on graphs and equations. 4. Analyze functions by investigating domain, range, rates of change, intercepts, zeros, asymptotes, and local and global behavior. ** 5. Describe the properties and features of any non-degenerate conic section from an equation or graph. 6. Describe and perform transformations, such as arithmetically combining, composing, and inverting commonly used functions using technology, to perform operations on more complicated symbolic expressions. 7. Relate the situation to the graph and the function values for direct, inverse, and joint variations. 8. Relate functions to their inverses and their reflections over the line y = x. 9. Write an equation for conic sections from a graph. 10. Analyze functions and their graphs for symmetries. 11. Use a variety of symbolic representations for functions and relations, including piecewise functions.
Grade 6 (E-F-G) Grade 7 (F-G-H) Grade 8 (G-H-I) Grade 9-10 (H-I-J) Grade 11-12 (I-J)		

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Mathematics Performance Descriptors

8C Students who meet the standard can solve problems using systems of numbers and their properties. *(Problem solving)*

Stage E	Stage F	Stage G
1. Solve problems with whole numbers using order of operations, equality properties, and appropriate field properties.	1. Identify and explain incorrect uses of the commutative, associative, and distributive properties. 2. Identify and provide examples of the identity property of addition and multiplication. 3. Identify and provide examples of inverse operations. 4. Explain why division by zero is undefined.	1. Solve arithmetic and linear equations using the properties of equality and inequality. 2. Identify and provide examples or counter examples as appropriate for the reflexive, symmetric and transitive properties of inequality.
Grade 6 (E-F-G) Grade 7 (F-G-H) Grade 8 (G-H-I) Grade 9-10 (H-I-J) Grade 11-12 (I-J)		

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Mathematics Performance Descriptors

8C Students who meet the standard can solve problems using systems of numbers and their properties. *(Problem solving)*

Stage H	Stage I	Stage J
1. Solve arithmetic and simple algebraic equations using properties of real numbers, equality and inequality, and justify the procedures. 2. Solve simple algebraic equations for a given variable using inverse operations.	1. Describe and compare the properties of linear and quadratic functions. ** 2. Solve problems by recognizing how an equation changes when parameters change. 3. Interpolate and extrapolate to solve problems using systems of numbers. 4. Solve problems using translations and dilations on basic functions.	1. Describe and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions. * 2. Identify and explain the relationship between arithmetic/geometric sequences and linear/exponential functions. 3. Describe the relationship of a mathematical model of a problem to the real problem. 4. Apply sequences and their properties to solve real problems. 5. Model and solve real problems using mathematical functions and relations. 6. Identify essential quantitative relationships in a situation and determine the class or classes of functions (e.g., power, exponential, logarithmic) that might model the relationships. ** 7. Explain and apply relationships of x, y, and t in parametric equations.
Grade 6 (E-F-G) Grade 7 (F-G-H) Grade 8 (G-H-I) Grade 9-10 (H-I-J) Grade 11-12 (I-J)		

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Mathematics Performance Descriptors

8D Students who meet the standard can use algebraic concepts and procedures to represent and solve problems. *(Connection of 8A, 8B, 8C to solve problems)*

Stage E	Stage F	Stage G
1. Create and solve linear equations involving whole numbers using a variety of methods (e.g., guess and check, bean stick counters).	1. Create, model, and solve algebraic equations using concrete materials. 2. Solve linear equations, including direct variation, with whole number coefficients and solutions using algebraic or graphical representations.	1. Solve simple linear equations, including direct variation, with integral coefficients using algebraic or graphical representations. 2. Solve simple problems involving quadratic relationships using technology for graphing.
Grade 6 (E-F-G) Grade 7 (F-G-H) Grade 8 (G-H-I) Grade 9-10 (H-I-J) Grade 11-12 (I-J)		

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Mathematics Performance Descriptors

8D Students who meet the standard can use algebraic concepts and procedures to represent and solve problems. *(Connection of 8A, 8B, 8C to solve problems)*

Stage H	Stage I	Stage J
1. Solve algebraic equations or word problems that involve linear equations or inequalities using algebraic or graphical representations. ** 2. Solve absolute value equations or inequalities in one variable using algebraic or graphical representations. 3. Create word problems that meet given conditions and represent linear relationships.	1. Solve equivalent forms of equations, inequalities, and systems of equations with fluency—mentally or with paper-and-pencil in simple cases and using technology in all cases. ** 2. Create word problems that meet given conditions and represent simple power or exponential relationships, or direct or inverse variation situations. 3. Solve simple quadratic equations using algebraic or graphical representations. 4. Solve problems of direct variation situations using a variety of methods.	1. Solve problems using linear programming. 2. Solve problems using equations of exponential and logarithmic growth. 3. Solve problems using direct, inverse, and mixed variation. 4. Apply solutions of real problems to similar situations with appropriate adaptation. 5. Solve problems using rational equations and inequalities. 6. Set up and solve problems of non-linear growth.
Grade 6 (E-F-G) Grade 7 (F-G-H) Grade 8 (G-H-I) Grade 9-10 (H-I-J) Grade 11-12 (I-J)		

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Mathematics Performance Descriptors

9A Students who meet the standard can demonstrate and apply geometric concepts involving points, lines, planes, and space. *(Properties of single figures, coordinate geometry and constructions)*

Stage E	Stage F	Stage G
1. Identify, compare, and analyze attributes of two- and three-dimensional shapes and develop vocabulary to describe the attributes. * 2. Classify two- or three-dimensional shapes according to their properties (e.g., regular and irregular, concave and convex, types of quadrilaterals, pyramids, and prisms). ** 3. Investigate and describe the results of subdividing and combining shapes. ** 4. Describe paths using coordinate systems. ** 5. Determine the distance between points along horizontal and vertical lines of a coordinate system. ** 6. Identify and justify rotational symmetry in two- and three-dimensional shapes. ** 7. Identify and describe how geometric figures are used in practical settings (e.g., construction, art, advertising, architecture). 8. Identify, sketch, and build two- and three-dimensional shapes given attribute clues. 9. Copy a line segment or an angle using a straightedge and a compass. 10. Construct a perpendicular bisector of a line segment.	1. Plot and read ordered pairs of numbers in all four quadrants. 2. Describe sizes, positions, and orientations of shapes under transformations, including dilations. 3. Perform simple constructions (e.g., equal segments, angle and segment bisectors, or perpendicular lines, inscribing a hexagon in a circle) with a compass and straightedge or a mira. 4. Determine and describe the relationship between pi, the diameter, the radius, and the circumference of a circle. **5.** Determine unknown angle measures using angle relationships and properties of a triangle or a quadrilateral.	1. Examine and describe a geometric shape, such as a regular polygon or a quadrilateral with pairs of parallel or perpendicular sides, using coordinate geometry. ** 2. Draw geometric shapes with specified properties, such as side lengths or angle measures. ** 3. Examine and describe line or rotational symmetry of objects in terms of transformations. 4. Draw transformations of figures in a plane to match specified criteria. 5. Perform constructions of congruent angles or parallel lines using a compass and straightedge, paper folding, or a mira. 6. Determine the relationship among the number of edges, faces, and vertices in a three-dimensional object.
Grade 6 (E-F-G) Grade 7 (F-G-H) Grade 8 (G-H-I) Grade 9-10 (H-I-J) Grade 11-12 (I-J)		

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Mathematics Performance Descriptors

9A Students who meet the standard can demonstrate and apply geometric concepts involving points, lines, planes, and space. *(Properties of single figures, coordinate geometry and constructions)*

Stage H	Stage I	Stage J
1. Represent and analyze the properties of geometric shapes using coordinate geometry. **	1. Describe and apply properties of a polygon or a circle in a problem-solving situation.	1. Analyze geometric situations using Cartesian coordinates and other coordinate systems such as navigational, polar, or spherical systems. **
2. Draw the image of an object after a combination of transformations.	2. Classify angle relationships for two or more parallel lines crossed by a transversal.	2. Represent transformations of an object in the plane using function notation and matrices.
3. Identify possible types of two- or three-dimensional figures that would match a set of given conditions.	3. Analyze geometric situations using Cartesian coordinates. **	3. Represent and describe with the language of geometry real-life objects, paths and regions in space.
4. Determine if a triangle is possible using side lengths and the triangle inequality.	4. Represent transformations of an object in the plane using sketches, coordinates, and vectors.	4. Apply properties of two- and three-dimensional models to solve problems.
5. Solve pictorial or word problems that involve geometric relationships within a single geometric shape or figure, including the Pythagorean theorem.	5. Design a net that will create a given figure when folded.	
6. Analyze the results of a combination of reflections, rotations, and translations of a figure, and determine alternate motions that could produce the same results.	6. Solve problems using constructions.	
7. Combine simple construction techniques to construct squares, equilateral triangles, or other simple combinations of equal segments, angles, etc.	7. Gain insights into, and answer questions in, other areas of mathematics using geometric models. **	
8. Analyze properties of a shape that enable it to tessellate the plane.	8. Calculate distance, midpoint coordinates, and slope using coordinate geometry.	
	9. Visualize a three-dimensional object from different perspectives and describe their cross sections. **	
	10. Identify and apply properties of medians, altitudes, angle bisectors, perpendicular bisectors, and midlines of a triangle.	
Grade 6 (E-F-G) Grade 7 (F-G-H) Grade 8 (G-H-I) Grade 9-10 (H-I-J) Grade 11-12 (I-J)		

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Mathematics Performance Descriptors

9B Students who meet the standard can identify, describe, classify and compare relationships using points, lines, planes, and solids. *(Connections between and among multiple geometric figures)*

Stage E	Stage F	Stage G
1. Demonstrate congruence of plane figures using transformations (translation, rotation, reflection). 2. Determine if two polygons are congruent using measures of angles and sides. 3. Match a front, right side, and top view drawing with a three-dimensional model built with cubes. **4.** Identify and describe the five regular polyhedra. **5.** Create regular and semi-regular tessellations using pattern blocks, other manipulatives, or technology to tile a plane.	1. Determine the relationships between the number of vertices or sides in a polygon, the number of diagonals, and the sum of its angles. 2. Solve problems that involve vertical, complementary, and supplementary angles. 3. Analyze quadrilaterals for defining characteristics. 4. Create a three-dimensional object from any two-dimensional representation of the object, including multiple views, nets, or technological representations.	1. Describe, classify, and justify relationships among types of two- and three-dimensional objects using their defining properties. 2. Solve problems using properties of polygons and circles. 3. Classify and order quadrilaterals according to their properties.
Grade 6 (E-F-G) Grade 7 (F-G-H) Grade 8 (G-H-I) Grade 9-10 (H-I-J) Grade 11-12 (I-J)		

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Mathematics Performance Descriptors

9B Students who meet the standard can identify, describe, classify and compare relationships using points, lines, planes, and solids. *(Connections between and among multiple geometric figures)*

Stage H	Stage I	Stage J
1. Create and analyze scale models using proportional reasoning. 2. Solve problems involving similar figures. 3. Examine the congruence or similarity of objects using transformations. ** 4. Analyze properties of a combination of shapes that enable them to tessellate the plane.	1. Solve problems using triangle congruence and similarity of figures. 2. Extend knowledge of plane figure relationships to relationships within and between geometric solids. 3. Identify relationships among circles, arcs, chords, tangents, and secants. 4. Solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture using geometric ideas. ** 5. Analyze and describe the transformations that lead to successful tessellations of one or more figures.	1. Solve problems using relationships between and among figures. 2. Represent and describe with the language of geometry intersections and cross sections of three-dimensional objects.
Grade 6 (E-F-G) Grade 7 (F-G-H) Grade 8 (G-H-I) Grade 9-10 (H-I-J) Grade 11-12 (I-J)		

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Mathematics Performance Descriptors

9C Students who meet the standard can construct convincing arguments and proofs to solve problems. *(Justifications of conjectures and conclusions)*

Stage E	Stage F	Stage G
1. Make and test conjectures about mathematical properties and relationships and develop logical arguments to justify conclusions. ** 2. Make and test conjectures about the results of subdividing and combining shapes. **	1. Make, test, and justify conjectures about various quadrilateral and triangle relationships, including the triangle inequality. 2. Justify the relationship between vertical angles. 3. Justify that the sum of the angles of a triangle is 180 degrees.	1. Create and critique arguments concerning geometric ideas and relationships, such as the number of diagonals in a polygon, or the formula for the sum of the interior angles of any polygon. ** 2. Justify the area formulas for triangles, parallelograms, and trapezoids based on the formula for the area of a rectangle. 3. Make and test conjectures about the relationships between side length and angle measure in various triangles and quadrilaterals. 4. Justify the properties of angles formed by parallel lines cut by a transversal using appropriate terminology.
Grade 6 (E-F-G) Grade 7 (F-G-H) Grade 8 (G-H-I) Grade 9-10 (H-I-J) Grade 11-12 (I-J)		

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Mathematics Performance Descriptors

9C Students who meet the standard can construct convincing arguments and proofs to solve problems. *(Justifications of conjectures and conclusions)*

Stage H	Stage I	Stage J
1. Create and critique arguments concerning geometric ideas and relationships, such as congruence, similarity, the Pythagorean relationship, or formulas for surface areas or volume of simple three-dimensional objects. **	1. Create and critique arguments concerning geometric ideas and relationships such as properties of circles, triangles and quadrilaterals.	1. Prove conjectures about geometric figures on the plane or in space using coordinate geometry.
2. Justify the simple construction methods used to produce angle bisectors, perpendicular lines, and equilateral triangles.	2. Develop a formal proof for a given geometric situation on the plane.	2. Extend the ideas of formal and informal proof to non-geometric situations.
3. Represent, solve, and explain numerical and algebraic relationships using geometric concepts.	3. Provide a counter-example to disprove a conjecture.	3. Develop formal and informal proofs for three-dimensional figures.
4. Provide examples or counter-examples to either illustrate or disprove conjectures about geometric characteristics.	4. Develop conjectures about geometric situations with and without technology.	
	5. Justify constructions using geometric properties.	
	6. Describe the difference between an inductive argument and a deductive argument.	

Grade 6 (E-F-G) Grade 7 (F-G-H) Grade 8 (G-H-I) Grade 9-10 (H-I-J) Grade 11-12 (I-J)

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Mathematics Performance Descriptors

9D Students who meet the standard can use trigonometric ratios and circular functions to solve problems.

Stage E	Stage F	Stage G
9D not applicable for Stage E.	**9D not applicable for Stage F.**	1. Analyze the relationship between sides of right triangles using the Pythagorean theorem. 2. Solve problems that involve the use of proportions and the Pythagorean theorem in similar right triangles with whole number side lengths.

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Mathematics Performance Descriptors

9D Students who meet the standard can use trigonometric ratios and circular functions to solve problems.

Stage H	Stage I	Stage J
1. Recognize Pythagorean Triples. 2. Identify the basic trigonometric ratios in terms of lengths of the sides of a right triangle and an acute angle. 3. Solve for missing side lengths using the trigonometric ratios in right triangles. 4. Determine and justify the side length relationships present in 45^o-45^o-90^o triangles and 30^o-60^o-90^o triangles. 5. Determine the ratio of lengths of sides of a right triangle with given measures for its acute angles using appropriate technologies.	1. Determine distances and angle measures using indirect measurement and properties of right triangles. **2.** Solve problems using 45^o-45^o-90^o and 30^o-60^o-90^o triangles.	1. Solve problems using the Law of Sines and Law of Cosines. 2. Relate vector representation and trigonometric functions. 3. Solve problems using vectors. 4. Relate circular functions, arcs, and radian measure to triangle trigonometry and degree measure. 5. Simplify expressions and solve problems using trigonometric identities. 6. Solve trigonometric equations using circular functions. 7. Rotate conic sections using trigonometric functions. 8. Identify key characteristics of graphs of trigonometric functions and their inverses. 9. Graph trigonometric functions using translations and dilations. 10. Graph a given trigonometric function using its characteristics (e.g., period, amplitude).
Grade 6 (E-F-G) Grade 7 (F-G-H) Grade 8 (G-H-I) Grade 9-10 (H-I-J) Grade 11-12 (I-J)		

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Mathematics Performance Descriptors

10A Students who meet the standard can organize, describe and make predictions from existing data. *(Data Analysis)*

Stage E	Stage F	Stage G
1. Represent given data using double bar graphs, double line graphs, and stem and leaf plots with and without technology. 2. Select an appropriate graph format to display given data. 3. Read, interpret, infer, predict, draw conclusions, and evaluate data from any graph. 4. Determine mean, median, mode, minimum value, maximum value, and range, and discuss what each does to help interpret a given set of data.	1. Construct, read, interpret, infer, predict, draw conclusions, and evaluate data from various displays, including circle graphs. ** 2. Recognize and explain misleading displays of data due to inappropriate intervals on a scale.	1. Construct, read, interpret, infer, predict, draw conclusions, and evaluate data from various displays, including box and whiskers plots. ** 2. Find, use, and interpret measures of center and spread, including interquartile range. ** 3. Construct an equivalent data representation given data in a different form. 4. Recognize potential bias in data collection methods or data presentation.
Grade 6 (E-F-G) Grade 7 (F-G-H) Grade 8 (G-H-I) Grade 9-10 (H-I-J) Grade 11-12 (I-J)		

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Mathematics Performance Descriptors

10A Students who meet the standard can organize, describe and make predictions from existing data. *(Data Analysis)*

Stage H	Stage I	Stage J
1. Construct, read, interpret, infer, predict, draw conclusions, and evaluate data from various displays, including histograms and scatter plots. ** 2. Determine the best measure of central tendency from mean, median, or mode. 3. Discuss how data can be manipulated to represent different points of view based on the use of different measures of central tendency and based on different graphical displays. 4. Discuss biased reporting of data and questions that should be asked when data is viewed. 5. Analyze graphical displays of data for possible misleading characteristics. 6. Make conjectures about the possible correlation between two characteristics of a sample on the basis of scatter plots of the data and approximate lines of fit. *	1. Describe the meaning of measurement data and categorical data, of univariate and bivariate data, and of the term variable. ** 2. Display a scatter plot, describe its shape, and determine regression coefficients, regression equations, and correlation coefficients for bivariate measurement data using technological tools. 3. Evaluate published reports that are based on data by examining the design of the study, the appropriateness of the data analysis, and the validity of conclusions. * 4. Analyze two-variable data for linear or quadratic fit. 5. Make decisions based on data, including the relationships of correlation and causation.	1. Describe the differences among various kinds of studies and which types of inferences can legitimately be drawn from each. ** 2. Recognize how linear transformations of univariate data affect shape, center, and spread. 3. Describe how sample statistics reflect the values of population parameters and use sampling distributions as the basis for informal inference. ** 4. Present results and conclusions from given data using basic statistics (e.g., measures of central tendencies, standard deviation). 5. Interpolate, extrapolate, and make predictions from given information. 6. Evaluate survey results for conformity to simple distributions.
Grade 6 (E-F-G) Grade 7 (F-G-H) Grade 8 (G-H-I) Grade 9-10 (H-I-J) Grade 11-12 (I-J)		

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Mathematics Performance Descriptors

10B Students who meet the standard can formulate questions, design data collection methods, gather and analyze data, and communicate findings. *(Data Collection)*

Stage E	Stage F	Stage G
1. Design investigations to address a question and consider how data-collection methods affect the nature of a data set. 2. Propose and justify conclusions and predictions that are based on data, and design studies to further investigate the conclusions or predictions. *	1. Gather data by conducting simple simulations. 2. Collect data over time with or without technology.	1. Select and use appropriate data gathering techniques. 2. Formulate new questions using conjectures, and plan new studies to answer them. **
Grade 6 (E-F-G) Grade 7 (F-G-H) Grade 8 (G-H-I) Grade 9-10 (H-I-J) Grade 11-12 (I-J)		

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Mathematics Performance Descriptors

10B Students who meet the standard can formulate questions, design data collection methods, gather and analyze data, and communicate findings. *(Data Collection)*

Stage H	Stage I	Stage J
1. Formulate a question, design a study to answer the question, and collect data. ** 2. Analyze potential methods of collecting information and decide which methods would produce the most reliable and accurate data. 3. Analyze instruments used for surveys for errors and bias. 4. Analyze potential experiments or simulations for errors and bias.	1. Describe the characteristics of well-designed studies, including the role of randomization in surveys and experiments. ** 2. Discuss informally different populations and sampling techniques. 3. Decide if a survey was "successful" in gathering intended data and justify the decision.	1. Explore the variability of sample statistics from a known population and construct sampling distributions using simulations.** 2. Describe how sample statistics reflect the values of population parameters and use sampling distributions as the basis for informal inference.** 3. Create a survey from a critical question and decide which sampling technique to use for the survey. 4. Evaluate surveys for clarity, bias, return rate, and specialized audiences.
Grade 6 (E-F-G) Grade 7 (F-G-H) Grade 8 (G-H-I) Grade 9-10 (H-I-J) Grade 11-12 (I-J)		

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Mathematics Performance Descriptors

10C Students who meet the standard can determine, describe and apply the probabilities of events. *(Probability, including counting techniques)*

Stage E	Stage F	Stage G
1. List all possible outcomes of compound, independent events (e.g., toss a coin and spin a spinner). 2. Assign a value of zero to probabilities that are impossible and a value of one to probabilities that are certain. 3. Express simple probabilities as a fraction between zero and one. 4. Predict the probability of outcomes of simple experiments and test the predictions. *	1. Record probabilities as fractions, decimals, or percents. 2. Demonstrate that the sum of all probabilities equals one. 3. Determine empirical probabilities from a set of data provided. 4. Set up a simulation to model the probability of a single event. 5. Discuss the effect of sample size on the empirical probability compared to the theoretical probability. 6. List outcomes by a variety of methods (e.g., tree diagram). 7. Determine theoretical probabilities of simple events.	1. Discuss odds versus probability. 2. Make and test conjectures about the results of experiments and simulations using proportionality and basic understanding of probability. ** 3. Compute probabilities for simple compound events using methods such as organized lists and tree diagrams. **
Grade 6 (E-F-G) Grade 7 (F-G-H) Grade 8 (G-H-I) Grade 9-10 (H-I-J) Grade 11-12 (I-J)		

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Mathematics Performance Descriptors

10C Students who meet the standard can determine, describe and apply the probabilities of events. *(Probability, including counting techniques)*

Stage H	Stage I	Stage J
1. Describe and explain complementary and mutually exclusive events using appropriate terminology. ** 2. Design and conduct experiments or simulations for probability, including the possible use of technology to simulate events. 3. Discuss the difference in empirical and theoretical probability. 4. Compute probabilities for simple compound events using a variety of methods, including area models. 5. Identify situations where dependent and independent events occur. 6. Determine probabilities using simple counting techniques. 7. Discuss situations where permutations and combinations should be used in counting outcomes.	1. Determine geometric probability based on area. 2. Calculate probability using Venn diagrams. 3. Determine simple probabilities using frequency tables. 4. Construct empirical probability distributions using simulations. ** 5. Describe the concepts of conditional probability. 6. Develop an understanding of permutations and combinations as counting techniques. *	1. Determine the theoretical probability for a chance event using the binomial probability model. 2. Describe the normal curve and use its properties to answer questions about sets of data that are assumed to be normally distributed. 3. Identify patterns from a sample space. 4. Describe a simulation for a more advanced experiment. 5. Carry out a simulation to estimate probabilities, and if possible, compare it to the theoretical probability. 6. Compute and interpret the expected value of random variables in simple cases. * 7. Apply advanced counting techniques to determine probability.
Grade 6 (E-F-G) Grade 7 (F-G-H) Grade 8 (G-H-I) Grade 9-10 (H-I-J) Grade 11-12 (I-J)		

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GLOSSARY

ACCEPTABLE LEVELS OF ACCURACY - the precision determined by the situation or the given numbers, students should help develop what is acceptable according to the situation.

ADDITIVE IDENTITY PROPERTY - the sum of any number and zero is the original number; zero is the identity element of addition.

ANALYZE - to break down into parts and explain or demonstrate the logic of a situation or a process.

ANGLE RULER - a hinged ruler with a protractor attached for reading the measure of an angle in degrees.

ASSOCIATIVE PROPERTY - property about grouping of numbers; of addition, the formula $(a + b) + c = a + (b + c)$.

ASYMPTOTE - a straight line always approaching but never intersecting a curve.

BOX PLOT - a representation of data above a numbered scale where the "box" encloses all data between the median of the lower half (quartile 1) and the median of the upper half (quartile 3), with a vertical line inside the box to indicate the median of the data; a dot represents each of the high and low values of the data, and a horizontal line called a whisker connects each dot to the box.

CALCULATOR NOTATION - the symbols used by a calculator for scientific notation.

CAPACITY - the amount of liquid that can fill an object.

CARTESIAN PLANE - a rectangular coordinate system consisting of a horizontal number line (x-axis) and a vertical number line (y-axis), intersecting at the origin (zero on each number line).

CHORD - a straight line joining any two points on an arc, curve or circle.

CLASS OF FUNCTIONS - family of functions such as linear, quadratic, power (polynomial), exponential, or logarithmic.

CLASSES OF NUMBERS - family of numbers or number systems such as natural, integer, rational, irrational, real, or complex.

COMMUTATIVE PROPERTIES - properties about order - of addition, $a + b = b + a$; of multiplication, $a \times b = b \times a$.

COMPATIBLE NUMBERS - numbers that can be easily manipulated and operated on mentally.

COMPLEX NUMBERS - numbers of the form a + bi, where a and b are real numbers and i = $\sqrt{-1}$ -1.

COMPOSE NUMBERS - put a set of numbers together to form a new number using addition or multiplication.

COMPOSITION - see "compose numbers".

COMPOUND EVENTS - two or more events in a probability situation such as flipping a coin and spinning a spinner.

CONCRETE MATERIALS - objects to be manipulated (e.g., pattern blocks, snap cubes, geoboards, tangrams, color tiles, base ten blocks).

CONIC SECTION - figures represented by cross-sections of a cone or two cones placed apex to apex - parabola, circle, ellipse, hyperbola.

CONSTANT RATE OF CHANGE - set of data or table of values in which the amount of the dependent variable changes by a constant (fixed) value as the value of the independent variable changes by a constant value.

CONSTANT RATIO - see constant rate of change.

CONSTRUCT - create a figure using only a straight edge and compass.

CONTRACTION - the opposite of dilation, a figure resulting from multiplying all dimensions of a given figure by a number between zero and one.

CONVERSION FACTOR - relationship between two units from different systems of measurement used to convert from one system to the other (e.g., 2.54 centimeters corresponds to 1 inch).

COORDINATE SYSTEM - set of ordered pairs used to locate an object or point on the two-dimensional plane.

COUNTING TECHNIQUES - a variety of methods used to determine the total possible outcomes, typically in a probability situation, including the multiplication principle, trees and lists.

DECOMPOSE NUMBERS - break up numbers into addends or factors.

DEPENDENT VARIABLE - a variable whose value depends on the value of another variable in combination with or without other numbers.

DERIVED MEASUREMENT - a measurement that is a combination of two other measurements such as speed in miles per hour.

DESCRIBE - to explain orally or in writing.

DEVELOP – to be involved in reasoning, exploration, conjecturing, using manipulatives or sketches to gain understanding of concepts or relationships.

DEVELOP FLUENCY - to become skillful in working with numbers, both in accuracy and speed.

DILATIONS - a figure resulting from multiplying all dimensions of a given figure by a number greater than one.

DIRECT VARIATION - a variation of the type y = kx; the graph is a straight line through the origin.

DISTRIBUTIVE PROPERTY - the property which relates multiplication and addition; the formula, a(b + c) = a x b + a x c.

DOUBLE LINE GRAPHS - graphs in which two sets of data are graphed at the same time, connecting each set with line segments.

DRAW - to create a figure using numbered scales on tools such as rulers and protractors.

ERROR TOLERANCE - the value allowable above and below a number or its approximation.

EXACT NUMBER - a numerical result that has not been rounded or estimated.

EXPLICIT FORM - a formula for any term of a sequence given the number of the term.

EXPONENTIAL FUNCTION - a function of the form $y = a(b^x)$.

EXPRESSION - combination of numerals or numerals and variables that indicate a finite number of operations, not an equation.

EXTRAPOLATE - to use given information to predict values beyond the set of given values using either a formula or a reasonable estimate.

FAMILIAR FRACTIONS - commonly used fractions such as halves, thirds, fourths, fifths, sixths, eighths and tenths.

FIELD PROPERTIES - closure for addition and multiplication, commutative for addition and multiplication, associative for addition and multiplication, identity for addition and multiplication, inverse for addition and multiplication, distributive for multiplication over addition.

FLEXIBLY - usually applied to computation, where students should be able to mentally manipulate numbers and components of numbers to create a solution to a problem.

FUNCTION RULE - the set of operations that describes the process that takes the independent variable and transforms it into the dependent variable in a consistent way.

GREATEST COMMON FACTOR - the greatest number that is a factor of each of the given numbers.

GROWING PATTERN – a pattern where the number of objects in the pattern increases from term to term.
GROWTH FORMULA – either a linear or exponential equation that describes the growth over time.

GROWTH PATTERN – a set of values usually visualized by plotting points on a grid and fitting either a linear or an exponential equation to the scatter plot.

IDENTIFY - to choose from a set or to name cases in which the desired result is present or true.

INDEPENDENT VARIABLE - in the equation, $y = 3x + 2$, x is the independent variable and y is the dependent variable. Substituting any value for x leads to a value for y.

INDIRECT MEASUREMENT - a measurement that is impossible or impractical to be measured directly or physically, usually calculated using a formula or a known relationship.

INDIRECT TECHNIQUE - the method used to determine an indirect measurement.

INTEGER - any whole number or its opposite.

INTEGRAL COEFFICIENT - in the expression, $3x$, 3 is the coefficient. Integral coefficients are coefficients that are integers.

INTERPOLATE - determine a value within a set of given values using a formula, rule, or reasonable estimation.

INVERSE OPERATION - examples of inverse operations are addition and subtraction, multiplication and division, extracting a root and raising to a power.

INVERSE VARIATION - a variation of the type $xy = k$; the graph is a hyperbola with axes as asymptotes.

IRREGULAR SHAPES - shapes that are not one of the named geometric shapes or combinations of shapes; shapes that may contain curved portions rather than straight-line segments.

JOINT VARIATION - variation in which the values of one variable depend on two or more direct variations, $z = kxy$.

JUSTIFY - give a logical explanation or informal proof of a mathematical situation, computation or property.

LAW OF COSINES - for any triangle with sides a, b, c, and angles A, B, and C, a squared = b squared + c squared - 2bc Cos A.

LAW OF SINES - for any triangle with sides a, b, c and angles A, B, and C, sin A/a = sin B/b = sin C/c.

LINE SYMMETRY - a figure has line symmetry if there is a least one line that divides the figure into two parts that are mirror images of each other.

LINEAR EQUATION - an equation which has a straight line graph.

LOCAL BEHAVIOR - a description of the values of a function or relation within a small interval of the independent variable.

LOGARITHM - a logarithm of a number is the exponent to which a given base must be raised to produce the given number.

LOGARITHMIC GROWTH - a set of values which are approximated by an equation of the form y = log b x.

LOGARITHMIC NOTATION - use of the symbols "log" or "ln" in context.

LOWER BOUND - any number below which a function value may approach but not pass.

MANIPULATIVES - objects that can be arranged, built, and moved around by hand, (see concrete materials.)

MATHEMATICAL NOTATION - correct use of labels, symbols, and abbreviations in a mathematics context.

MIRA - a plastic device which is used to determine and complete symmetries by reflecting images and allowing the user to also see through the reflecting surface.

MIXED VARIATION - variation that contains both direct and inverse variation.

MODE - the most frequently occurring value or values in a set of values.

MODEL - to create, using concrete materials, drawings or symbols; a representation of a mathematical relationship or situation.

NATURAL NUMBER - the counting numbers, starting with one.

NAVIGATIONAL SYSTEM - compass directions or bearings in a variety of formats.

NET - a two-dimensional pattern that will fold into a three-dimensional figure.

NON-CONSTANT RATE OF CHANGE - set of data or table of values in which the amount of the dependent variable does not change by a constant value as the value of the independent variable changes by a constant value.

NON-DEGENERATE CONIC SECTIONS - see conic sections.

NON-REGULAR SHAPE - a shape that does not have all sides congruent and all angles congruent.

ORDER - to place numbers in order from smallest to largest or largest to smallest.

ORDER OF OPERATIONS - the rule for using operations on numbers; first parentheses, then exponents, then multiplication and division, then addition and subtraction.

PARAMETRIC EQUATIONS - two equations which express the coordinates of x and y as separate functions of a common variable, called the parameter that is usually time.

PERPENDICULAR BISECTOR - a line segment or ray that intersects a segment at its midpoint at a 90^{o} angle.

PIECEWISE FUNCTION - a function consisting of two or more equations, defined for specified intervals of the independent variable.

POLYHEDRON - a solid figure bounded by faces that are polygons; two faces meet at an edge, two or more edges meet at a vertex.

PRIME FACTORIZATION - the unique set of factors of a number, all of which are prime numbers.

PROBABILITY - a ratio of the number of desired outcomes and the number of possible outcomes.

PROBABILITY EVENTS - the set of all possible outcomes of an experiment is the sample space; any subset of the sample space is an event.

PROPERTIES OF EQUALITY - reflexive property (a = a), symmetric property (if a = b, then b = a), and transitive property (if a = b and b = c, then a = c).

PROPORTIONALITY - a relationship described by a constant ratio.

PROVE - use logical arguments, definitions, theorems, and properties to show that a relationship is true for all numbers or specific set of figures.

PYTHAGOREAN THEOREM - In a right triangle with sides a and b, and hypotenuse c, the theorem states that $c^2 = a^2 + b^2$.

PYTHAGOREAN TRIPLES - sets of three numbers that satisfy the Pythagorean theorem.

QUADRATIC EQUATION - any equation of the form $y = ax^2 + bx + c$.

RADIAN - an angle measure based on the premise that there are 2π radians in a complete circle.

RATIO - a comparison of one quantity to another.

RATIO OF SIMILITUDE - the simplest form ratio of the measures of corresponding parts of similar figures.

REAL NUMBERS - rational and irrational numbers.

RECOMPOSE - put addends or factors of a given number back together in a way different from the original arrangement or decomposition.

RECURSIVE FORM - a formula for the next term of a sequence given the term before it.

REFLECTION - a rigid motion of an object on the plane, also known as a "flip", where the object is replicated as if it were folded across a line and imprinted in its new location.

REGULAR POLYGON - a polygon with equal sides and equal angles.

REGULAR SHAPE - see regular polygon.

RIGID MOTION – a motion that preserves shape and size.

ROTATIONAL SYMMETRY - a figure has rotational symmetry if it can be rotated (turned) less than 360 degrees about a point so that it appears the same as the original figure.

ROTATION - a rigid motion in the plane in which a figure is rotated (turned) about a point in the plane.

SCIENTIFIC NOTATION - a notation for expressing very large and very small numbers as a product of a decimal number greater than or equal to one and less than ten and a power of ten.

SET OF DATA POINTS - data collected and placed into ordered pairs for the purpose of graphing.

SIMPLE POLYGONS - convex, closed shapes bounded by line segments joined end to end.

SIMPLE EVENTS - a single activity in a probability experiment such as flipping a coin.

SIMULATION - an experiment that has the same number of outcomes as a given situation but is easier or more practical to carry out than the given situation.

SLOPE - the ratio of the increase in the y-values to the increase in the x-values between any two ordered pairs.

SLOPE-INTERCEPT FORM - the form of a linear equation $y = mx + b$ where m represents the slope and b represents the y-intercept.

STANDARD FORM - the form of a number expressed as a sum of products involving powers of ten.

STEM AND LEAF PLOTS - a method of displaying data where the leading digit(s) are the stem and the ending single digits are arranged in ascending order to the side representing the leaves.

SUCCESSIVE APPROXIMATION - a sequence of approximations, each one closer to the desired value.

SURFACE AREA - the total area required to cover the surface of a three-dimensional object.

SYSTEM OF EQUATIONS - a set of two or more equations.

TABLE OF VALUES - a table of two columns, the first representing values of the independent variable, the second representing the values of the dependent variable.

TESSELLATION - a tiling of a plane with no gaps and no overlaps.

TRANSFORMATIONS - motions in the plane; translations, rotations, reflections, dilations and contractions.

TRANSLATION - a rigid motion of an object in the plane also known as a "slide" where the object moves from one location to another.

TREE DIAGRAM - a counting technique where outcome options are shown as "branches" and the total number of branches at the end of the tree represent the total number of possible outcomes.

TRIANGLE INEQUALITY - the property that states that the length of any side of a triangle is less than the sum of the lengths of the other two sides.

UNIT ANALYSIS - the process of using conversion factors to change from one measure or rate to another.

UNIT OF ATTRIBUTE - the unit chosen depends upon the attribute being measured (e.g., for the attribute "length", a unit might be "meters").

UNIT FRACTION – fraction with one as a numerator and a natural number as a denominator.

UPPER BOUND - any number above which a function value may approach but not pass.

VECTOR - a line segment with both direction and length.

VERTICAL ANGLES - non-adjacent angles formed by the intersection of two lines.

VERTICES - plural form of vertex; the point of intersection of the rays of an angle, "corner" point of any geometric figure bounded by lines, planes, or lines and planes.

VOLUME - the amount of space occupied in three dimensions.

RELATIONSHIP OF PERFORMANCE DESCRIPTORS TO NATIONAL AND STATE STANDARDS

The writing team began its work by examining the National Council of Teachers of Mathematics new publication, *Principles and Standards for School Mathematics*. Since this document had just been released after an intensive national review period, the writers agreed that it was a credible source on which to base the Illinois Performance Descriptors. Many of NCTM's standards were adapted to meet the needs of the Illinois Performance Descriptors and are designated by **. Also, statements that were quoted directly are identified with an *.

The writers also had other states' documents to consult as their work progressed. The documents used were from Colorado, Kansas, Nevada, North Carolina, Texas, and Virginia. The ACT *Standards for Transition* were also examined and incorporated where applicable.

Because of the variety of works reviewed, the writers feel that they have produced a comprehensive description of mathematics performance for Illinois' students.

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