

# Mathematics

## [27.130]

### STANDARD 1

**The competent teacher of mathematics communicates mathematical content and concepts.**

**Knowledge Indicators** - *The competent teacher of mathematics:*

- 1A. understands the dynamics of working collaboratively with others.
- 1B. understands learning styles and learning strategies.

**Performance Indicators** - *The competent teacher of mathematics:*

- 1C. communicates verbally and in written, visual, and symbolic forms using appropriate technology.
- 1D. creates effective learning environments where students will be able to work collaboratively in one-to-one, small-group, and large-group contexts.
- 1E. analyzes the thinking and learning strategies of all students to extend mathematical knowledge.

### STANDARD 2

**The competent teacher of mathematics develops and utilizes a variety of problem-solving techniques.**

- 2A. **Knowledge Indicator** - The competent teacher of mathematics understands the many strategies for problem solving.

**Performance Indicators** - *The competent teacher of mathematics:*

- 2B. uses problem explorations and modeling to extend mathematical knowledge of all students.
- 2C. generalizes results of problems and extends them to other problem situations.

### STANDARD 3

**The competent teacher of mathematics recognizes a variety of reasoning techniques and applies appropriate techniques to concepts, procedures, and conjectures.**

- 3A. **Knowledge Indicator** - The competent teacher of mathematics understands various ways of reasoning with respect to concepts, procedures, and conjectures.

**Performance Indicators** - *The competent teacher of mathematics:*

- 3B. applies mathematical reasoning and appropriate technologies in the development of concepts, procedures, and conjectures.
- 3C. generalizes reasoning skills within the study of mathematics and applies or extends them to other contexts.

## STANDARD 4

**The competent teacher of mathematics makes connections within and among the various branches of mathematics and other disciplines.**

**Knowledge Indicators** - *The competent teacher of mathematics:*

- 4A. understands the connections within the mathematics curriculum.
- 4B. understands mathematical connections to school curriculums and to other disciplines.
- 4C. has knowledge of the historical development of mathematics that includes contributions of men and women from various cultures.

**Performance Indicators** - *The competent teacher of mathematics:*

- 4D. develops the connections within and among the various branches of mathematics.
- 4E. connects mathematics to other disciplines.

## STANDARD 5

**The competent teacher of mathematics selects, integrates and uses appropriate technologies.**

**Knowledge Indicators** - *The competent teacher of mathematics:*

- 5A. becomes familiar with the capabilities and benefits of current and emerging technologies.
- 5B. understands the selection, integration, and utilization of appropriate technologies throughout the mathematics curriculum.

**Performance Indicators** - *The competent teacher of mathematics:*

- 5C. selects appropriate technologies for instruction.
- 5D. integrates current technology as appropriate for instruction.

## STANDARD 6

**The competent teacher of mathematics demonstrates, applies, and explains a knowledge and sense of number.**

- 6A. **Knowledge Indicators** - The competent teacher of mathematics knows and understands the concepts of number, number theory, and numeration systems.

The early childhood or elementary school teacher:

- 6A1. understands number sense, including concepts of order, magnitude, mental math, estimation, and place value, and senses the reasonableness of results.
- 6A2. understands decimals, fractions, ratios, proportions, and percents as ways to represent numbers and relationships.

- 6A3. knows the properties of prime and composite numbers.
- 6A4. The middle school teacher knows and understands the content outlined in subsection 6A(1-3) of this Section and has knowledge of complex numbers, modular systems, and matrices.
- 6A5. The secondary school mathematics teacher knows and understands the content outlined in subsections 6A (1-3) and 6A4 of this Section and understands polar and vector representations of complex numbers, algebraic structures, and topics from discrete mathematics.
- 6B. **Knowledge Indicators** - The competent teacher of mathematics knows and understands algorithms of operations.
- 6B1. The early childhood or elementary school teacher understands composition, decomposition, and recomposition of numbers, including place value of any base, factors and multiples, prime and composite numbers, inverses, and the application of these concepts.
- 6B2. The middle school teacher knows and understands the content outlined in subsection 6B1 of this Section, understands operations with algebraic expressions, and has knowledge of symbolic manipulators.
- 6B3. The secondary school mathematics teacher knows and understands the content outlined in subsections 6B1 and 6B2 of this Section and has knowledge of iterative processes as they relate to fractals and other applications.
- 6C. **Performance Indicators** - The competent teacher of mathematics applies the concepts of number theory and numeration systems.
- The early childhood or elementary school teacher:
- 6C1. uses number sense to judge the reasonableness of results.
- 6C2. applies proportional reasoning to solve problems.
- 6C3. uses the properties of prime and composite numbers to determine greatest common factors and least common multiples to solve problems.
- 6C4. performs operations in any base and converts between bases.
- 6C5. The middle school teacher demonstrates the competence outlined in subsection 6C (1-4) of this Section and computes and interprets the results of computation using complex numbers, modular systems, and matrices.
- 6C6. The secondary school mathematics teacher demonstrates the competence outlined in subsections 6C (1-4) and 6C5 of this Section and computes using polar and vector representations of complex numbers.
- 6D. **Performance Indicators** - The competent teacher of mathematics models, explains, develops, and applies algorithms of operations.
- 6D1. The early childhood or elementary school teacher chooses appropriately from mental math, paper and pencil, manipulative, and technology to perform computations.
- 6D2. The middle school teacher demonstrates the competence outlined in subsection 6D1 of this Section and models, develops, and applies algorithms with technology.
- 6D3. The secondary school mathematics teacher demonstrates the competence outlined in subsections 6D1 and 6D2 of this Section and uses numerical approximations as a basis for numerical integration and numerical-based proofs.

## STANDARD 7

**The competent teacher of mathematics understands and applies the process of measurement.**

- 7A. **Knowledge Indicators** - The competent teacher of mathematics knows customary, metric, and nonstandard measurement.

The early childhood or elementary school teacher:

- 7A1. knows how to measure length, area, volume, capacity, time, temperature, angles, weight, and mass.  
7A2. has knowledge of non-standard and emerging units such as graphic screen pixels and font size.

The middle school teacher knows and understands the content outlined in subsection 7A(1-2) of this Section and:

- 7A3. understands trigonometric ratios and relationships.  
7A4. understands how changing one measure of a multi-dimensional object may affect other measures.  
7A5. understands conversion factors as they apply to dimensional analysis.

7A6. The secondary school mathematics teacher knows and understands the content outlined in subsections 7A(1-2) and 7A(3-5) of this Section and understands nonlinear scales.

- 7B. **Performance Indicators** - The competent teacher of mathematics selects and applies appropriate units and tools for measuring, comparing, and ordering.

The early childhood or elementary school teacher:

- 7B1. uses customary, metric, and nonstandard measures.  
7B2. selects and uses appropriate tools to perform various measurements.  
7B3. estimates measurement.  
7B4. determines acceptable measures of accuracy and calculates relative error.

7B5. The middle school teacher demonstrates the competence outlined in subsection 7B(1-4) of this Section and uses trigonometric ratios and their relationships to solve problems.

7B6. The secondary school mathematics teacher demonstrates the competence outlined in subsections 7B(1-4) and 7B5 of this Section and applies nonlinear scales (e.g., exponential and logarithmic).

- 7C. **Performance Indicators** - The competent teacher of mathematics uses formulas and other procedures for computing or estimating the measure of multidimensional objects.

The early childhood or elementary school teacher:

- 7C1. measures objects using both direct and indirect measurement.  
7C2. uses formulas to aid in indirect measurement.  
7C3. applies measurements and formulas to irregular shapes, regions, and solids.  
7C4. reads and interprets scale drawings.

The middle school teacher demonstrates the competence outlined in subsection 7C(1-4) of this Section and

- 7C5. explains how changing one measure of a multi-dimensional object affects other measurements.
- 7C6. reads and interprets topographical maps and architectural drawings.

The secondary school mathematics teacher demonstrates the competence outlined in subsections 7C(1-4) and 7C(5-6) of this Section and:

- 7C7. applies dimensional analysis.
- 7C8. uses modeling and visualization to hypothesize about and predict measurements.

## **STANDARD 8**

**The competent teacher of mathematics demonstrates, applies and explains a knowledge and sense of algebra via patterns, functions, symbols, and models.**

- 8A. **Knowledge Indicators** - The competent teacher of mathematics knows and understands applications of concepts, representations, and relationships of variables and patterns.

The early childhood or elementary school teacher:

- 8A1. understands the order of operations.
- 8A2. understands the concept of variables.
- 8A3. understands symbolic, numeric, and graphical representations of mathematical situations.
- 8A4. understands the properties and operations of real numbers in an algebraic context.
- 8A5. The middle school teacher knows and understands the content outlined in subsection 8A(1-4) of this Section and understands a wide range of modeling applications involving graphs, tree charts, and other visual representations of data with multiple dimensions.
- 8A6. The secondary school mathematics teacher knows and understands the content outlined in subsections 8A(1-4) and 8A5 of this Section and understands variable relationships, logic, and concepts of discrete mathematics.

- 8B. **Knowledge Indicators** - The competent teacher of mathematics knows and understands concepts and representations of relations and functions.

The early childhood or elementary school teacher:

- 8B1. understands the symbolic, numeric, graphical, and verbal representations of relations and functions and the relationships among them.
- 8B2. understands polynomial and exponential functions.
- 8B3. The middle school teacher knows and understands the content outlined in subsection 8B(1-4) of this Section and understands logarithmic, parametric, trigonometric, rational, radical, and absolute value relations and their graphs.
- 8B4. The secondary school mathematics teacher knows and understands the content outlined in subsections 8B(1-4) and 8B3 of this Section and understands linear algebra.

- 8C. **Performance Indicators** - The competent teacher of mathematics knows and understands the development and applications of calculus.

The early childhood or elementary school teacher:

- 8C1. understands concepts of rates of change and patterns that lead to limits.
- 8C2. understands concepts of distance, area, volume, and shapes that lead to limits.

The middle school teacher knows and understands the content outlined in subsection 8C(1-2) of this Section and:

- 8C3. knows the historical development of calculus.
- 8C4. understands the concept of limits.
- 8C5. understands the basic concepts of calculus and their applications.

8C6. The secondary school mathematics teacher knows and understands the content outlined in subsections 8C(1-2) and 8C(3-5) of this Section and understands the concepts of calculus and analytical geometry and their applications.

- 8D. **Knowledge Indicators** - The competent teacher of mathematics knows and understands algebraic structures.

The early childhood or elementary school teacher:

- 8D1. understands the properties of the real number systems.
- 8D2. understands the properties of sets and Venn diagrams.

8D3. The middle school teacher knows and understands the content outlined in subsection 8D(1-2) of this Section and understands the properties of complex and modular systems and their applications.

8D4. The secondary school mathematics teacher knows and understands the content outlined in subsections 8D(1-2) and 8D3 of this Section and understands groups, rings, integral domains, and fields.

- 8E. **Performance Indicators** - The competent teacher of mathematics applies concepts, representations, and relationships of variables and patterns.

The early childhood or elementary school teacher:

- 8E1. applies the order of operations to numerical and algebraic expressions.
- 8E2. identifies, completes, and extends patterns and sequences.
- 8E3. represents mathematical situations symbolically, numerically, and graphically.
- 8E4. applies properties and operations of real numbers in an algebraic context.
- 8E5. solves number sentences involving variables.
- 8E6. translates word situations to algebraic sentences and solves them using algebra.
- 8E7. uses equations and inequalities as a means of solving practical applications.

The middle school teacher demonstrates the competence outlined in subsection 8E(1-7) of this Section and:

- 8E8. uses a wide range of modeling applications involving graphs and tree charts.
- 8E9. solves systems of linear equations graphically and algebraically (including matrices and determinants).
- 8E10. solves systems of nonlinear equations and inequalities algebraically and graphically.

The secondary school mathematics teacher demonstrates the competence outlined in subsections 8E(1-7) and 8E(8-10) of this Section and:

- 8E11. explains and applies symbolic logic.
- 8E12. explains and applies induction and recursion.
- 8E13. explains and applies sequences and series.

- 8F. **Performance Indicators** - The competent teacher of mathematics uses concepts and representations of relations and functions and their applications.

The early childhood or elementary school teacher:

- 8F1. represents functions and relations in symbolic, numeric, graphical, and verbal forms.
- 8F2. finds and uses slopes and intercepts to construct, analyze, and interpret graphs of equations and inequalities.

The middle school teacher demonstrates the competence outlined in subsection 8F(1-2) of this Section and:

- 8F3. finds and uses slope, symmetry, roots, intercepts, critical points, and vertices to construct and interpret graphs of functions and relations.
- 8F4. recognizes and uses the equations of lines, hyperbolas, parabolas, circles, ellipses, and nonlinear equations.
- 8F5. formulates, explains, and solves problems involving nonlinear equations such as variation and exponential and logarithmic growth and decay.
- 8F6. The secondary school mathematics teacher demonstrates the competence outlined in subsections 8F(1-2) and 8F(3-6) of this Section and applies principles and properties of linear algebra.

- 8G. **Performance Indicators** - The competent teacher of mathematics at the middle school or secondary level explains and uses the concepts of calculus and applications.

The middle school teacher:

- 8G1. calculates and interprets basic limits.
- 8G2. illustrates the basic concepts of calculus using concrete applications.
- 8G3. The secondary school mathematics teacher demonstrates the competence outlined in subsection 8G(1-2) of this Section and uses differentiation, integration, and other concepts of calculus to solve problems and interpret results.

## STANDARD 9

**The competent teacher of mathematics knows and uses geometric: methods to analyze, categorize, and draw conclusions about points, lines, planes, and space.**

- 9A. **Knowledge Indicator** - The competent teacher of mathematics understands terminology, properties of two- and three-dimensional shaper, and the relationships among them.
- 9B. **Knowledge Indicator** - The competent teacher of mathematics knows Euclidean and non-Euclidean geometry, coordinate geometry, graph theory, and transformational geometry and the relationships among them.

9C. **Knowledge Indicators** - The competent teacher of mathematics understands the process of conjecturing, justifying, and proof.

9C1. The early childhood or elementary school teacher understands inductive and deductive reasoning.

9C2. The middle school teacher understands inductive and deductive reasoning and understands the appropriate uses of different types of proof.

9C2. The secondary school mathematics teacher knows and understands the content outlined in subsection 9C1 and 9C2 of this Section and extends the understanding of proof to finite and non-Euclidean settings.

9D. **Performance Indicators** - The competent teacher of mathematics uses and applies the properties of geometry.

The early childhood or elementary school teacher:

9D1. recognizes relationships and patterns in geometric figures.

9D2. uses characteristics of geometric figures including symmetry, congruence, and similarity to recognize, identify, build, draw, describe, analyze, and categorize two- and three-dimensional figures and tessellation.

9D3. applies geometric concepts to solve practical applications.

9D4. explains the relationships between perimeter, area, and volume of similar figures.

9D5. The middle school teacher demonstrates the competence outlined in subsection 9D(1-4) of this Section and uses trigonometry to solve practical applications.

9D6. The secondary school mathematics teacher demonstrates the competence outlined in subsections 9D(1-4) and 9D5 of this Section.

9E. **Performance Indicators** - The competent teacher of mathematics identifies, analyzes, categorizes, and applies multi-dimensional figures using spatial visualization skills and modeling.

The early childhood or elementary school teacher:

9E1. translates between two- and three-dimensional representations of the same figure including the use of coordinate geometry and graph theory.

9E2. uses manipulative, Euclidean geometry, coordinate geometry, transformational geometry, and appropriate technology to model mathematical concepts and solve problems.

The middle school teacher demonstrates the competence outlined in subsection 9E(1-2) of this Section and:

9E3. generates solids of revolution from two-dimensional figures.

9E4. gives examples of non-Euclidean geometry.

9E5. applies recursion and iteration geometrically.

9E6. recognizes and uses relationships that exist between algebra and geometry.

The secondary school mathematics teacher demonstrates the competence outlined in subsections 9E(1-2) and 9E(3-6) of this Section and:

9E7. describes relationships of the planar sections of three-dimensional objects.

9E8. explains relationships that exist between transformations (including matrix representations) as a geometric equivalence of the function concept.

- 9F. **Performance Indicators** - The competent teacher of mathematics constructs convincing arguments and proofs.
- 9F1. The early childhood or elementary school teacher makes and identifies mathematical conjectures and provides justification to support or refute conjectures using manipulatives; constructions; algebraic, coordinate, and transformational methods; interactive technology; and paragraph and two-column proofs.
- 9F2. The middle school teacher demonstrates the competence outlined in subsection 9G1 of this Section, constructs inductive, deductive, and indirect arguments and explains the difference among them.
- 9F3. The secondary school mathematics teacher demonstrates the competence outlined in subsections 9G1 and 9G2 of this Section and uses a formal axiomatic system to construct and analyze proofs.

## STANDARD 10

**The competent teacher of mathematics demonstrates knowledge and application of probability, descriptive and inferential statistics.**

- 10A. **Knowledge Indicators** - The competent teacher of mathematics understands how to collect, organize, and represent data to answer questions.
- The early childhood or elementary school teacher:
- 10A1. understands how to create tables, graphs, charts, pictures, and other visual representations of a set of data.
- 10A2. understands simple random sampling and recognizes bias.
- 10A3. understands line of best fit.
- 10A4. The middle school teacher knows and understands the content outlined in subsection 10A(1-3) of this Section and understands data trends and curves of best fit.
- 10A5. The secondary school mathematics teacher knows and understands the content outlined in subsections 10A(1-3) and 10A4 of this Section.
- 10B. **Knowledge Indicators** - The competent teacher of mathematics understands summary statistics.
- The early childhood or elementary school teacher:
- 10B1. understands measures of central tendency, variation, and position.
- 10B2. understands common distributions.
- 10B3. The middle school teacher knows and understands the content outlined in subsection 10B(1-2) of this Section.
- 10B4. The secondary school mathematics teacher knows and understands the content outlined in subsection 10B(1-2) and 10B3 of this Section and understands additional measures of central tendency and variation.

- 10C. **Knowledge Indicators** - The competent teacher of mathematics understands how to interpret and predict the results of data analysis.

The early childhood or elementary school teacher:

- 10C1. understands the design of observations and experiments to answer questions.  
10C2. understands the concept of reliability and validity.
- 10C3. The middle school teacher knows and understands the content outlined in subsection 10C(1-2) of this Section and understands correlation and regression techniques.
- 10C4. The secondary school mathematics teacher knows and understands the content outlined in subsections 10C(1-2) and 10C3 of this Section and understands the design of experiments used in testing hypotheses.

- 10D. **Knowledge Indicators** - The competent teacher of mathematics understands counting techniques and probability.

The early childhood or elementary school teacher:

- 10D1. understands fundamental counting principles, combinations, and permutations.  
10D2. understands probability and odds of events.  
10D3. understands dependent and independent events.
- 10D4. The middle school teacher knows and understands the content outlined in subsection 10D(1-3) and of this Section and understands the use of random variables to solve problems.
- 10D5. The secondary school mathematics teacher knows and understands the content outlined in subsections 10D(1-3) and 10D4 of this Section and understands the link between probability theory and inferential statistics.

- 10E. **Performance Indicators** – The competent teacher of mathematics poses questions and collects, organizes, and represents data to answer those questions.

The early childhood or elementary school teacher:

- 10E1. creates tables, graphs, charts, pictures, and other visual representations of a set of data.  
10E2. collects simple random samples and recognizes sample bias.  
10E3. uses visual techniques for finding, interpreting, and applying the line of best fit.
- 10E4. The middle school teacher demonstrates the competence outlined in subsection 10E(1-3) of this Section and finds and applies appropriate curves of best fit using technology.
- 10E5. The secondary school mathematics teacher demonstrates the competence outlined in subsections 10E(1-3) and 10E4 of this Section.

- 10F. **Performance Indicators** - The competent teacher of mathematics calculates, explains, and interprets summary statistics.

The early childhood, elementary, or middle school teacher:

- 10F1. uses measures of central tendency and variation to describe a set of data.  
10F2. uses common distributions as appropriate to solve problems.
- 10F3. The secondary school mathematics teacher demonstrates the competence outlined in subsection 10F(1-2) of this Section and uses additional measures of central tendency and variation to describe a set of data.

- 10G. **Performance Indicators** - The competent teacher of mathematics predicts, calculates, interprets, and applies the results of data analysis.

The early childhood, elementary, or middle school teacher:

- 10G1. develops a hypothesis based on a question or problem of interest and devises a plan for the collection of data.
- 10G2. uses simple survey and sampling techniques.
- 10G3. collects, records, organizes, displays, summarizes, and interprets data.
- 10G4. The secondary school mathematics teacher demonstrates the competence outlined in subsection 10G(1-3) of this Section and chooses an appropriate experimental design, selects and performs proper research procedures, and interprets results.

- 10H. **Performance Indicators** - The competent teacher of mathematics demonstrates and uses counting techniques and probability.

The early childhood or elementary teacher:

- 10H1. uses the fundamental counting principles, combinations, and permutations.
- 10H2. determines the probability and odds of events.
- 10H3. analyzes problem situations (e.g., fairness of games, lotteries).
- 10H4. creates simulations to analyze simple theoretical or experimental probabilities.
- 10H5. computes probabilities for dependent and independent events.

The middle school teacher demonstrates the competence outlined in subsection 10H(1-5) of this Section and:

- 10H6. determines probabilities involving combinations and permutations.
- 10H7. generates and interprets probability distributions for random variables.
- 10H8. The secondary school mathematics teacher demonstrates the competence outlined in subsections 10H(1-5) and 10H(6-7) of this Section and links probability to inferential statistics.

## STANDARD 11

**The competent teacher of mathematics understands the process of reading and demonstrates instructional abilities to teach reading in the content area of mathematics.**

**Knowledge Indicators** - *The competent teacher of mathematics:*

- 11A. understands that the reading process is the construction of meaning through the interactions of the reader's background knowledge and experiences, the information in the text, and the purpose of the reading situation.
- 11B. recognizes the relationships among the four language arts (reading, writing, listening, and speaking), and knows how to provide opportunities to integrate these through instruction.
- 11C. understands how to design, select, modify and evaluate materials in terms of the reading needs of the learner.
- 11D. understands the importance of and encourages the use of literature for adolescents in the curriculum and for independent reading.
- 11E. understands the relationship between oral and silent reading.
- 11F. understands the role of subject-area vocabulary in developing reading comprehension.
- 11G. understands the importance of the unique study strategies required of the specific content area in developing reading comprehension.
- 11H. understands the importance of the relationship between assessment and instruction in planning.

**Performance Indicators - *The competent teacher of mathematics:***

- 11I. plans and teaches lessons for students that develop comprehension of content-area materials through instructional practices that include analyzing critically, evaluating sources, and synthesizing and summarizing material.
- 11J. plans and teaches lessons on how to monitor comprehension and correct confusions and misunderstandings that arise during reading.
- 11K. plans and models the use of comprehension strategies before, during, and after reading of text.
- 11L. provides opportunities for students to develop content-area vocabulary through instructional practices that develop connections and relationships among words, use of context clues, and understanding of connotative and denotative meaning of words.
- 11M. plans and teaches lessons that encourage students to write about the content read in order to improve understanding.
- 11N. plans and teaches lessons to help students develop study strategies that include previewing and preparing to read text effectively, recognizing organizational patterns unique to informational text, and using graphic organizers as an aid for recalling information.
- 11O. plans and teaches units that require students to carry out research or inquiry using multiple texts, including electronic resources.
- 11P. provides continuous monitoring of students' progress through observations, work samples, and various informal reading assessments.
- 11Q. analyzes and evaluates the quality and appropriateness of instructional materials in terms of readability, content, length, format, illustrations, and other pertinent factors.
- 11R. promotes the development of an environment that includes classroom libraries that foster reading.